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Rheology of ice in glaciers and polar ice sheets: experimental results and modelling

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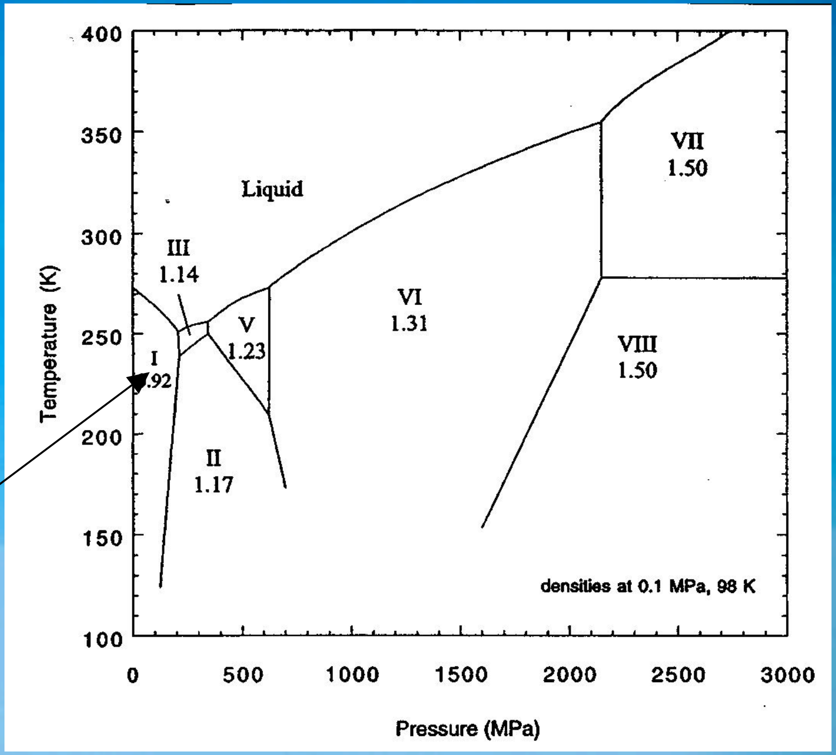
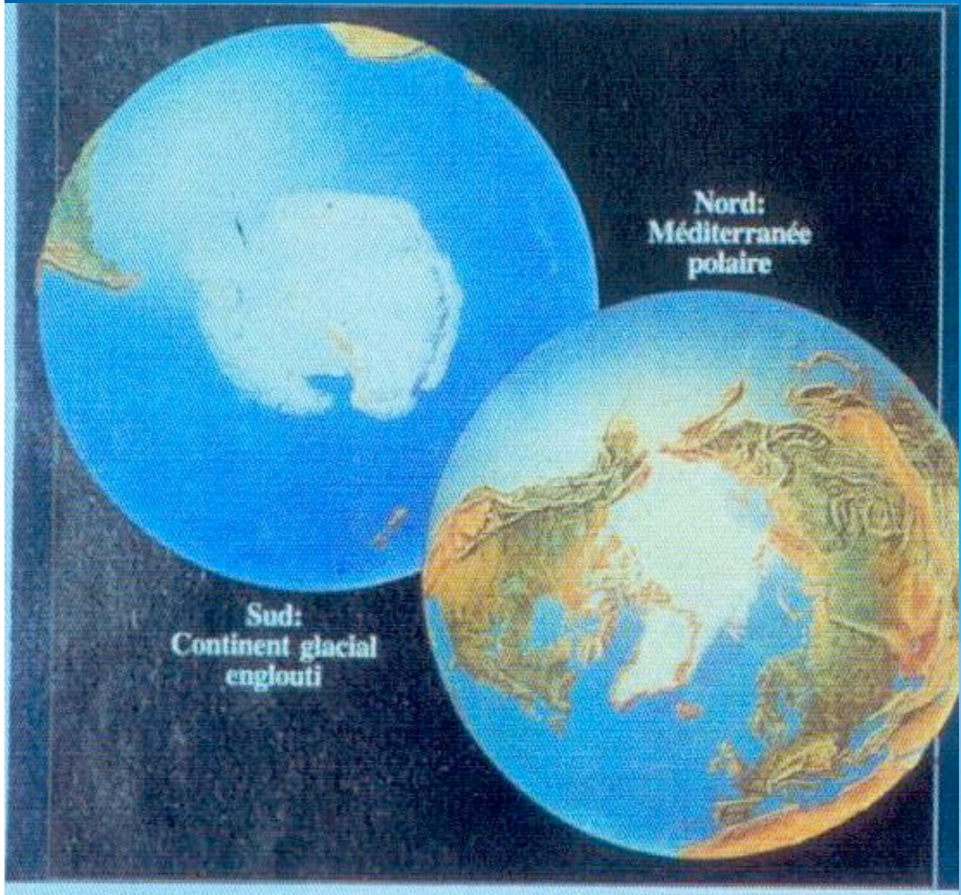
PHASE DIAGRAM OF ICE

Ice Ih *is the only "natural" phase on the Earth*

The ice pressure in polar ice sheets is lower than 40 MPa and temperature is higher than 210K



ICE ON THE EARTH



- Antarctic ice sheet: $14x10^6$ km^2
- Greenland: $1.8x10^6$ km^2
- Mountain glaciers: $0.35x10^6$ km^2

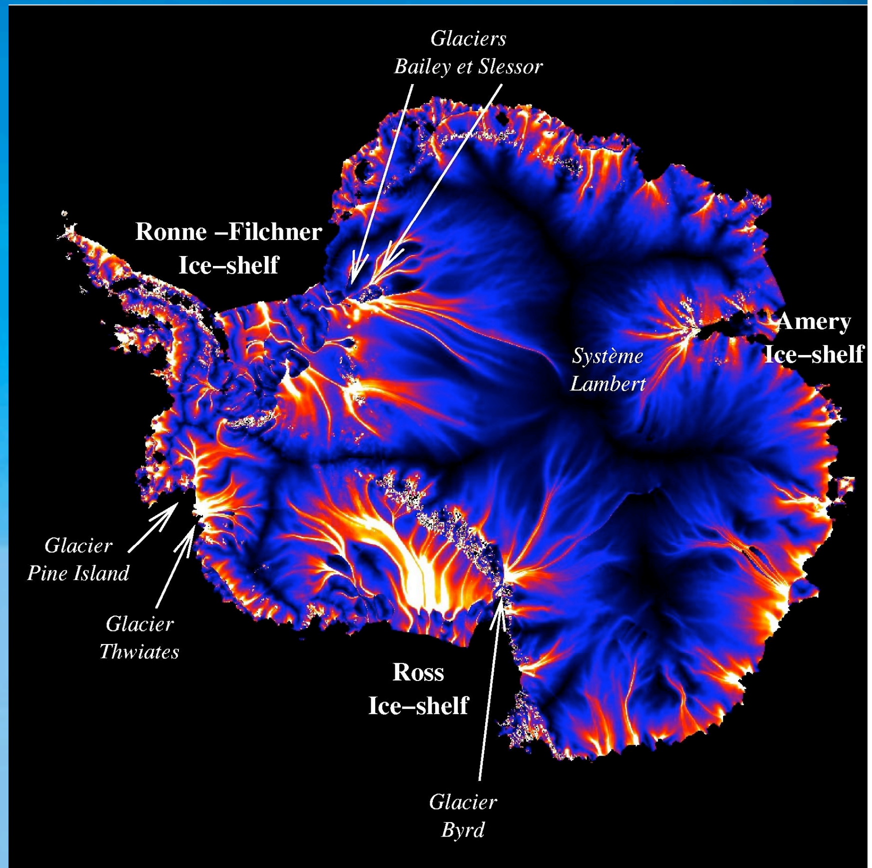
-Sea ice: $12x10^6$ km^2 in the North Hemisphere in winter

Mean thickness:
- *Antarctic ice sheet: 2,400m*
- *Greenland: 1,500 m*

THE ANTARCTIC ICE SHEET

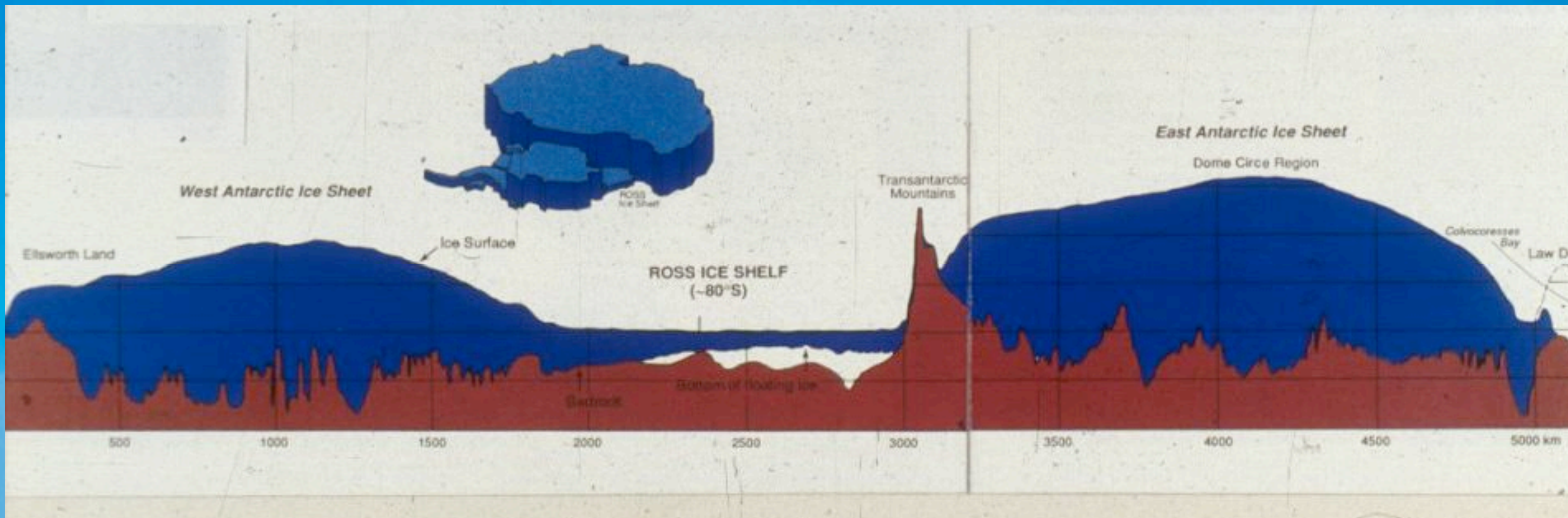
The surface velocities are between
1 and 1000 m/year

Strain rates are typically between 10^{-10} and 10^{-13} s^{-1}



From Rémy, 2003

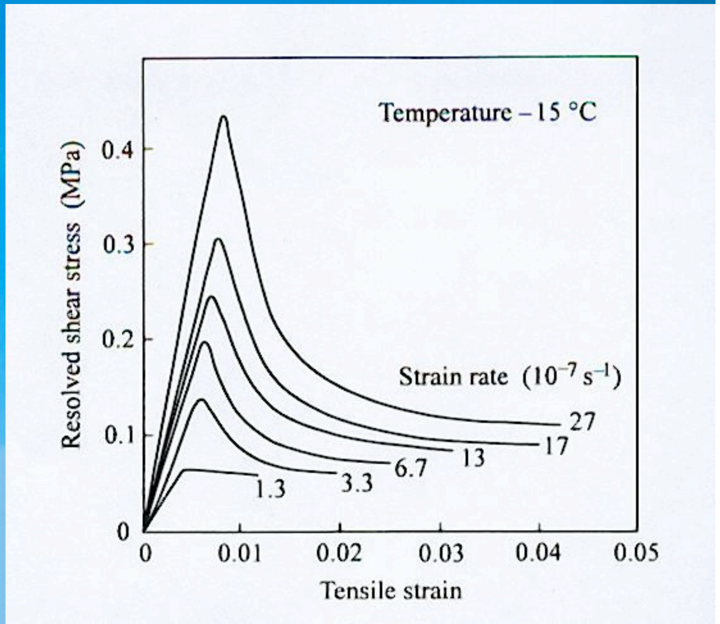
Vertical section of the Antarctic ice sheet



Viscoplastic deformation of the ice crystal

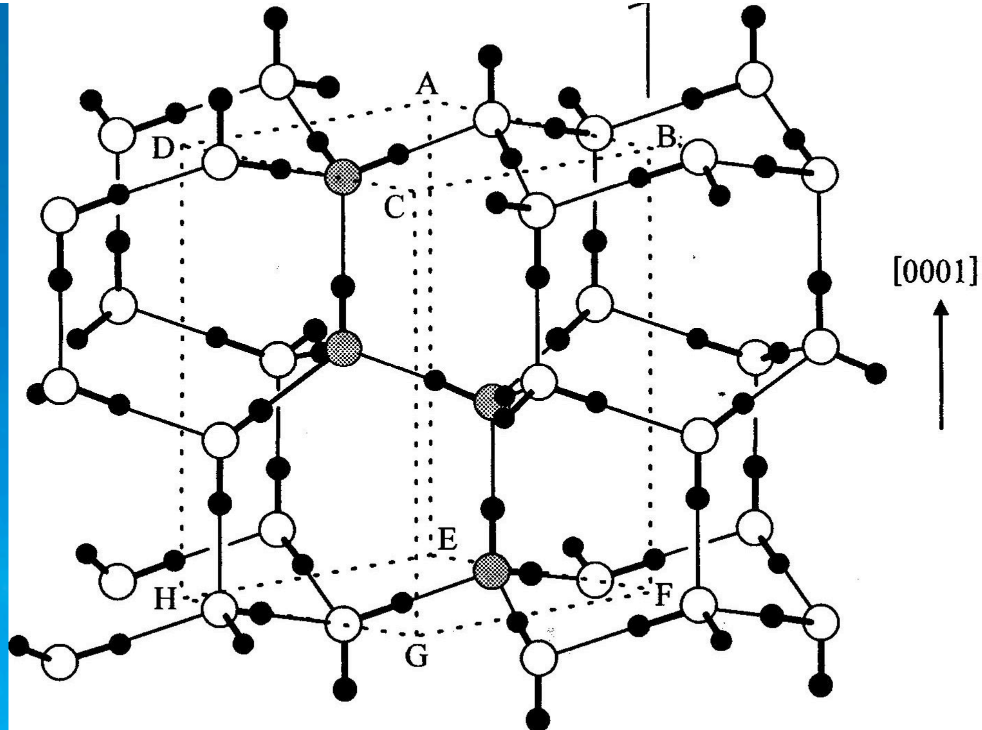
STRESS-STRAIN CURVES FOR BASAL SLIP

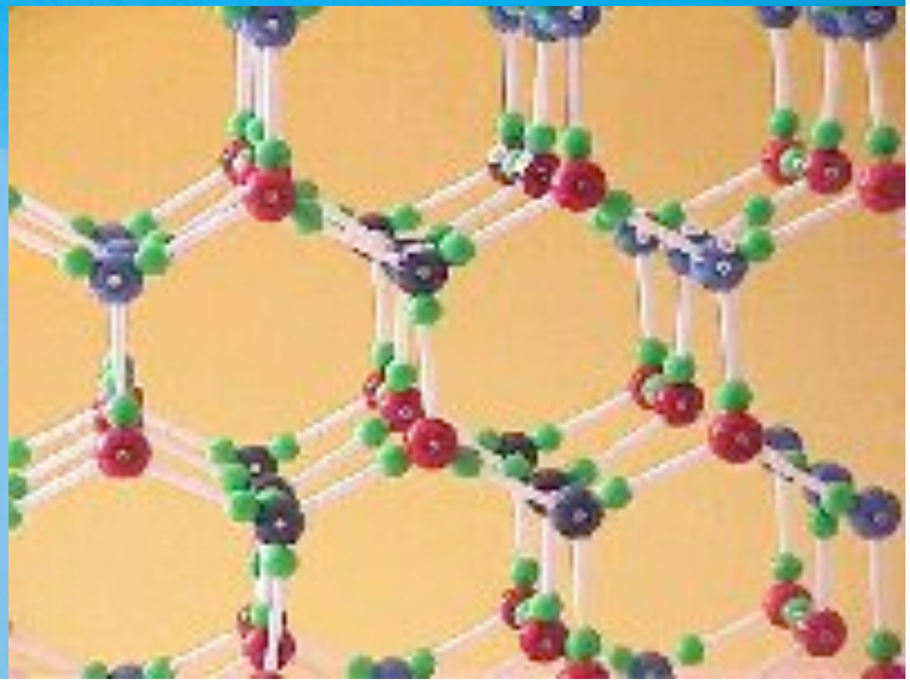
The peak stress and yield drop are related to the increase with strain of the mobile dislocation density (to be compared with semiconductors)



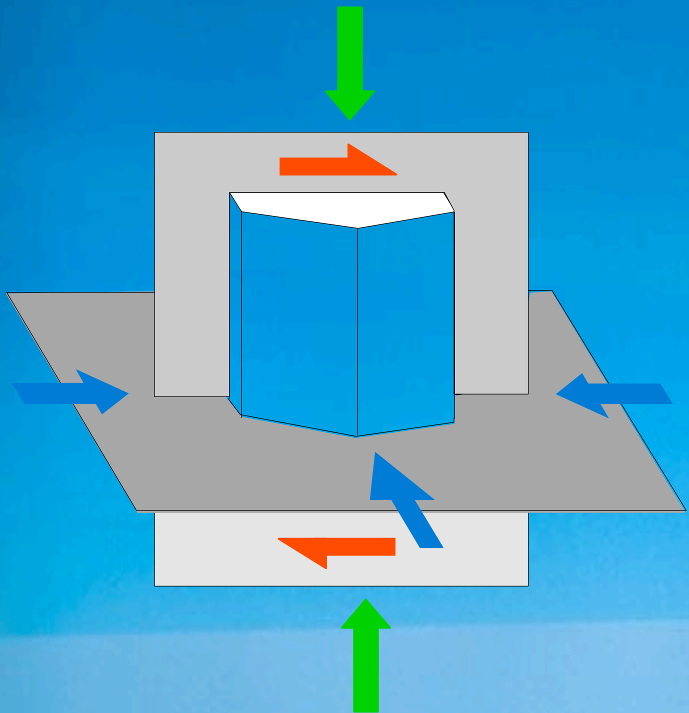
From Higashi, 1964

THE HEXAGONAL STRUCTURE OF ICE Ih





Plastic deformation of the ice crystal

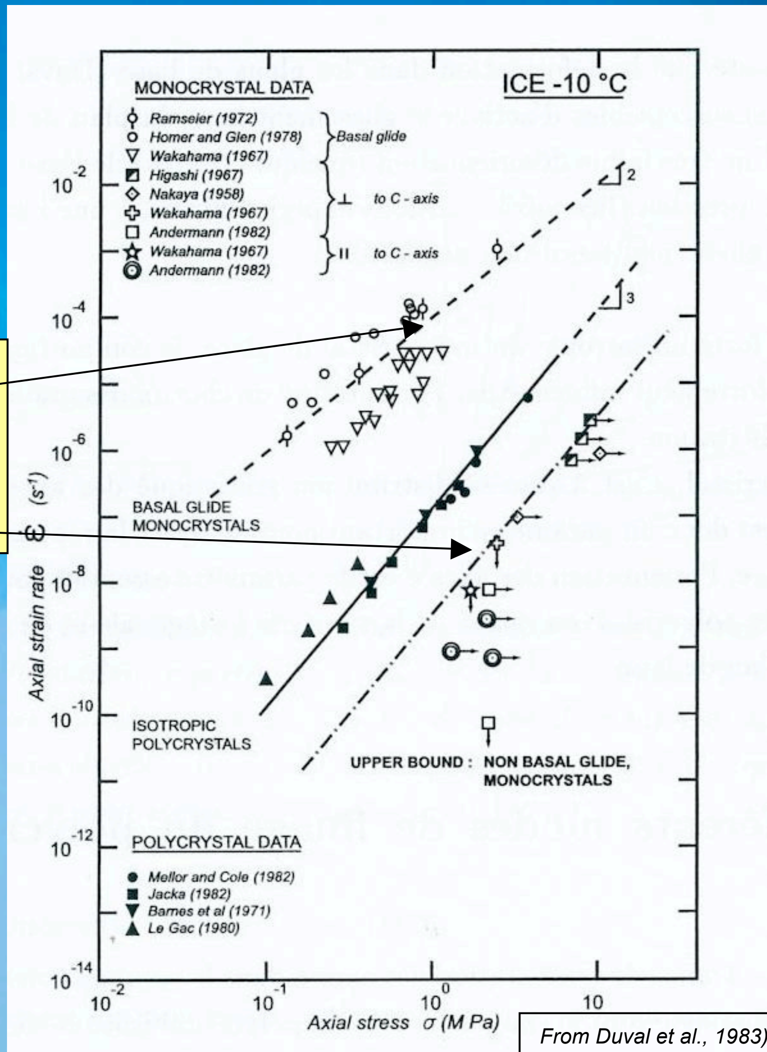


-shear on the basal plane is more than 1000 times faster than compression along basal plane or compression along the normal to the basal plane

The ice crystal deforms by the glide of basal dislocations on the basal plane

Basal and non-basal slip

For a given stress, strain rate for basal slip is more than 3 orders of magnitude higher than that for non-basal slip



From Duval et al., 1983)

The viscous behavior of the ice crystal

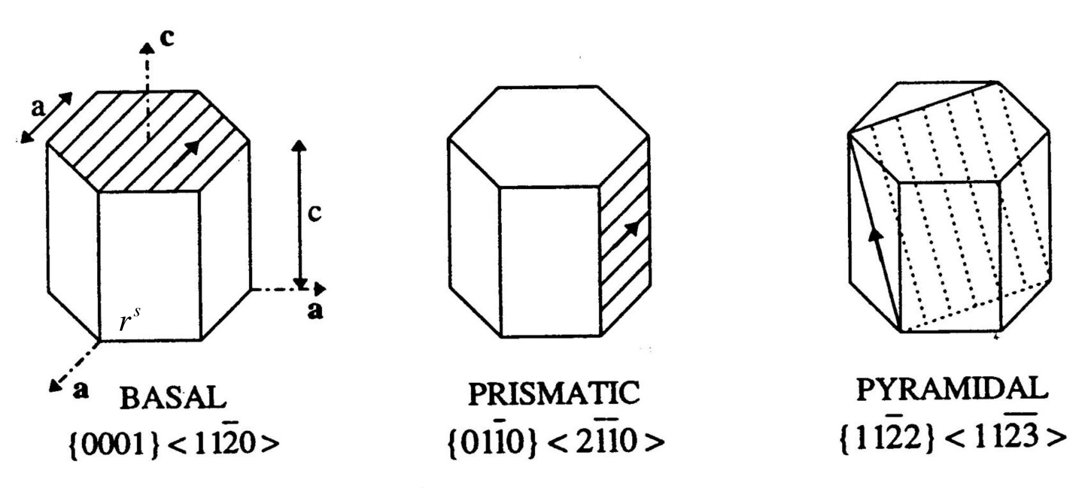
⇒The ice crystal can be considered as a transversely isotropic viscoplastic medium *(isotropy in the basal plane) with three grain rheology parameters:*

- the viscosity for shear parallel to the basal plane $\eta_{//}$
- the viscosity for shear in the basal plane $\eta_{//b}$
- the viscosity in compression (or tension) along the *c*-axis $\eta_{\perp}$

$\eta_{//}$ << *and* $\eta_{//b} \approx \eta_{\perp}$

Deformation behavior of the ice crystal from crystallographic slip

The shear strain rate on the slip system *s, is:* $\dot{\gamma}_s = \dot{\gamma}_o \left| \frac{\tau^s}{\tau_o^s} \right|^{n_s - 1} \left(\frac{\tau^s}{\tau_o^s} \right)$



τ_0^s is the critical stress for the slip system *s* and n_s is the exponent of the flow law associated with the slip system

τ_s is the shear stress on the slip system *s*

$$\dot{\underline{\varepsilon}}^g = \sum_s r^s \dot{\gamma}_s$$

r^s *is the Schmid tensor*

DISLOCATIONS AND SLIP SYSTEMS IN ICE Ih

The self-energy of a dislocation is given by:

$$E_d = \frac{Kb^2}{4\pi} \ln\left(\frac{\alpha R}{b}\right)$$

b is the Burgers vector, R is the outer cut-off radius, K is the energy factor expressed by the elastic constants

Table 2.1: Dislocations and slip systems in ice (from Hondoh, 2000)

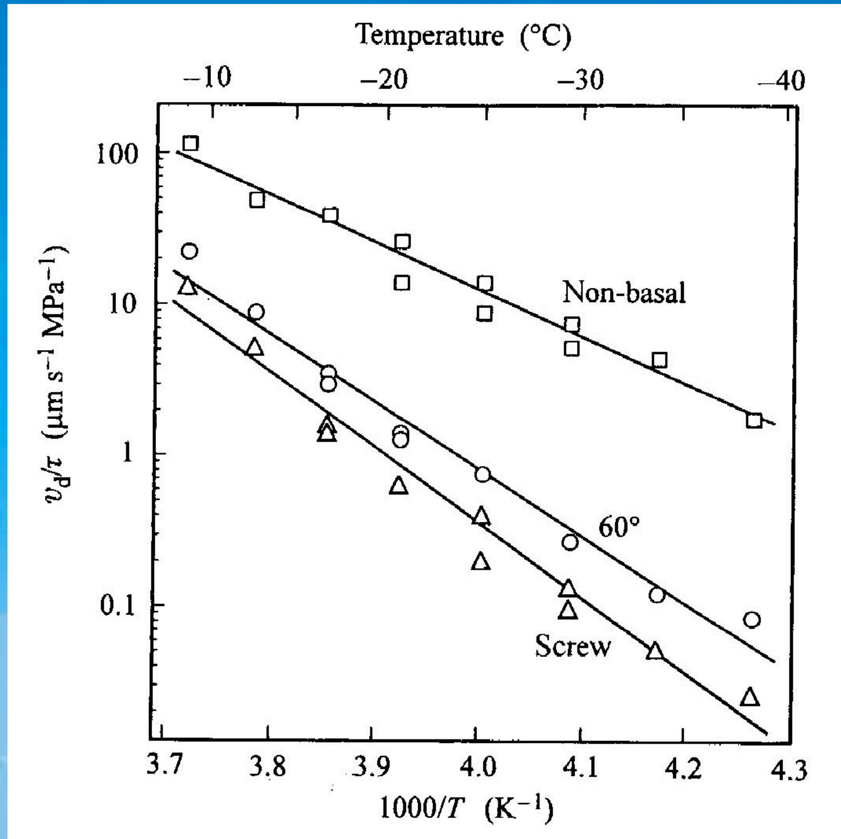
Type of dislocation	Burgers Vector	Relative self-energy	Slip plane	Fault energy (mJ/m^2)	Extended Width (nm)
Perfect	$1/3\langle 11\bar{2}0 \rangle$	1	(0001) or $(10\bar{1}0)$	-	-
	$\langle 0001 \rangle$	2.7	$(10\bar{1}0)$		
	$1/3\langle 11\bar{2}3 \rangle$	3.6	$(10\bar{1}1)$ or $(11\bar{2}2)$		
Partial	$1/3\langle 10\bar{1}0 \rangle$	0.33	(0001)	~ 0.6	24 (screw) 47 (60°) 55 (edge)
	$1/3\langle 0001 \rangle$	0.66	-	~ 0.9	>100
	$1/6\langle 20\bar{2}3 \rangle$	1	-	~ 0.3	>400

From Hondoh, 2000

Owing to the the low energy of stacking faults lying on the basal plane, dislocations should be dissociated

Velocity of non-interacting dislocations

The large variation in the mobility between basal and non-basal dislocations would be related to the level of the Peierls barrier



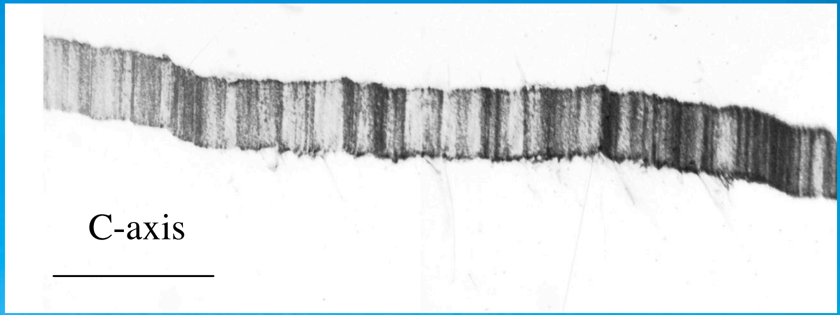
From Petrenko and Whitworth, 1999

OBSERVATION OF BASAL SLIP BANDS BY SYNCROTRON X-RAY TOPOGRAPHY

The slip pattern appears to be of fractal character with a scale invariance; as a consequence, the distance between slip bands has no meaning

↓

This result indicates long-range dislocations interactions

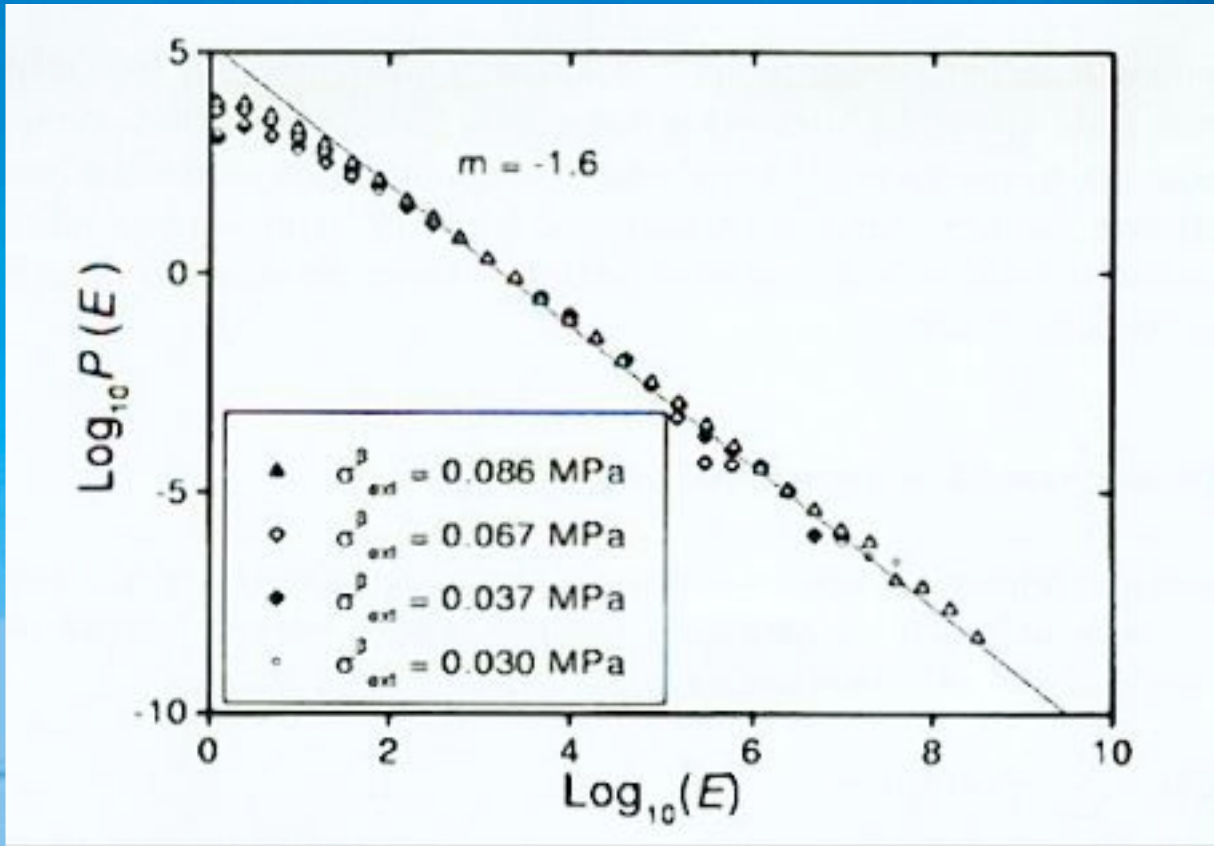


From Montagnat et al. 2006

Distribution of acoustic energy bursts recorded during creep tests of ice single crystals

Dislocation motion proceeds in avalanches involving the collective motion of many interacting dislocations

The distribution of acoustic emission amplitudes gives information on the size distribution of dislocation avalanches



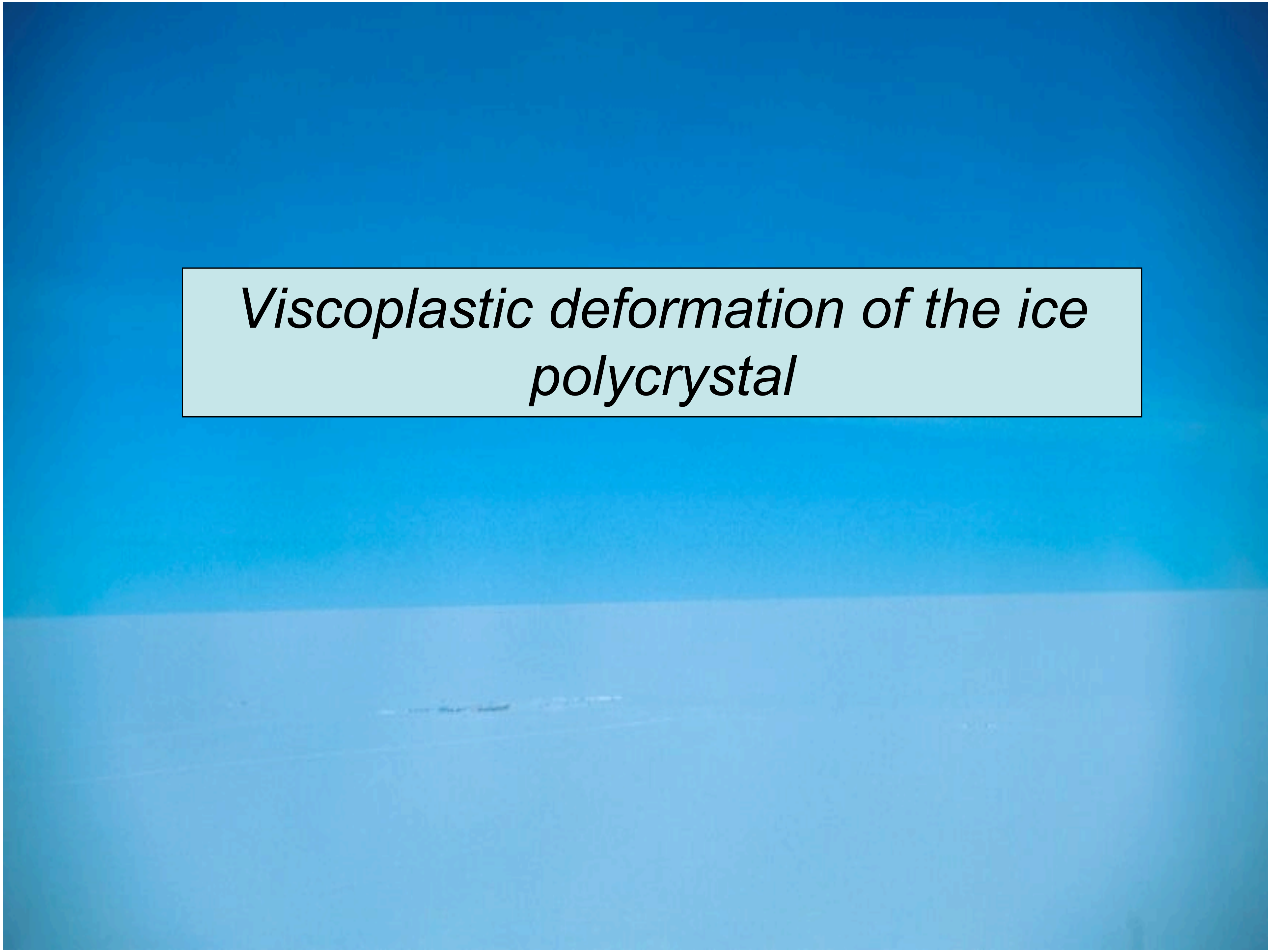
Over several orders, data follow a power law $P(E) \sim E^{-1.6}$

From Miguel et al., 2001

The rate-controlling processes in ice single crystals

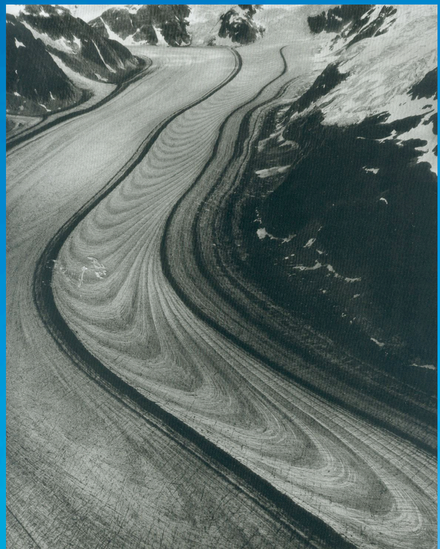
- Dislocation multiplication occurs by the cross slip of basal screw dislocations

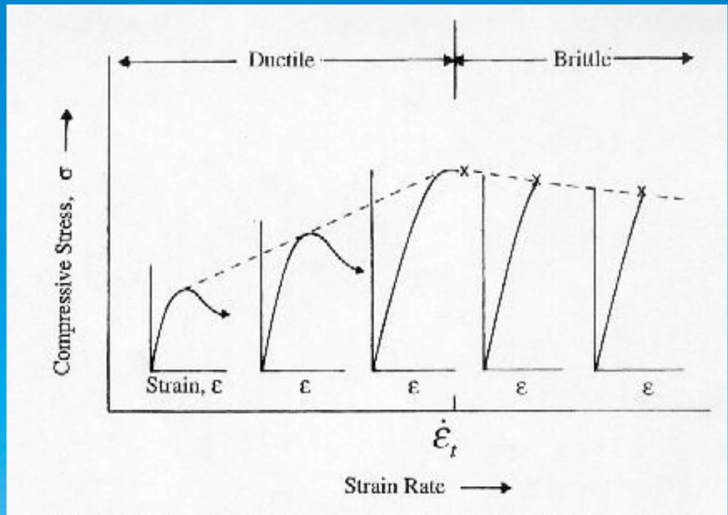
- The cross slip process should control the deformation of the single crystal



Viscoplastic deformation of the ice polycrystal

Polycrystalline ice: a brittle and viscous material

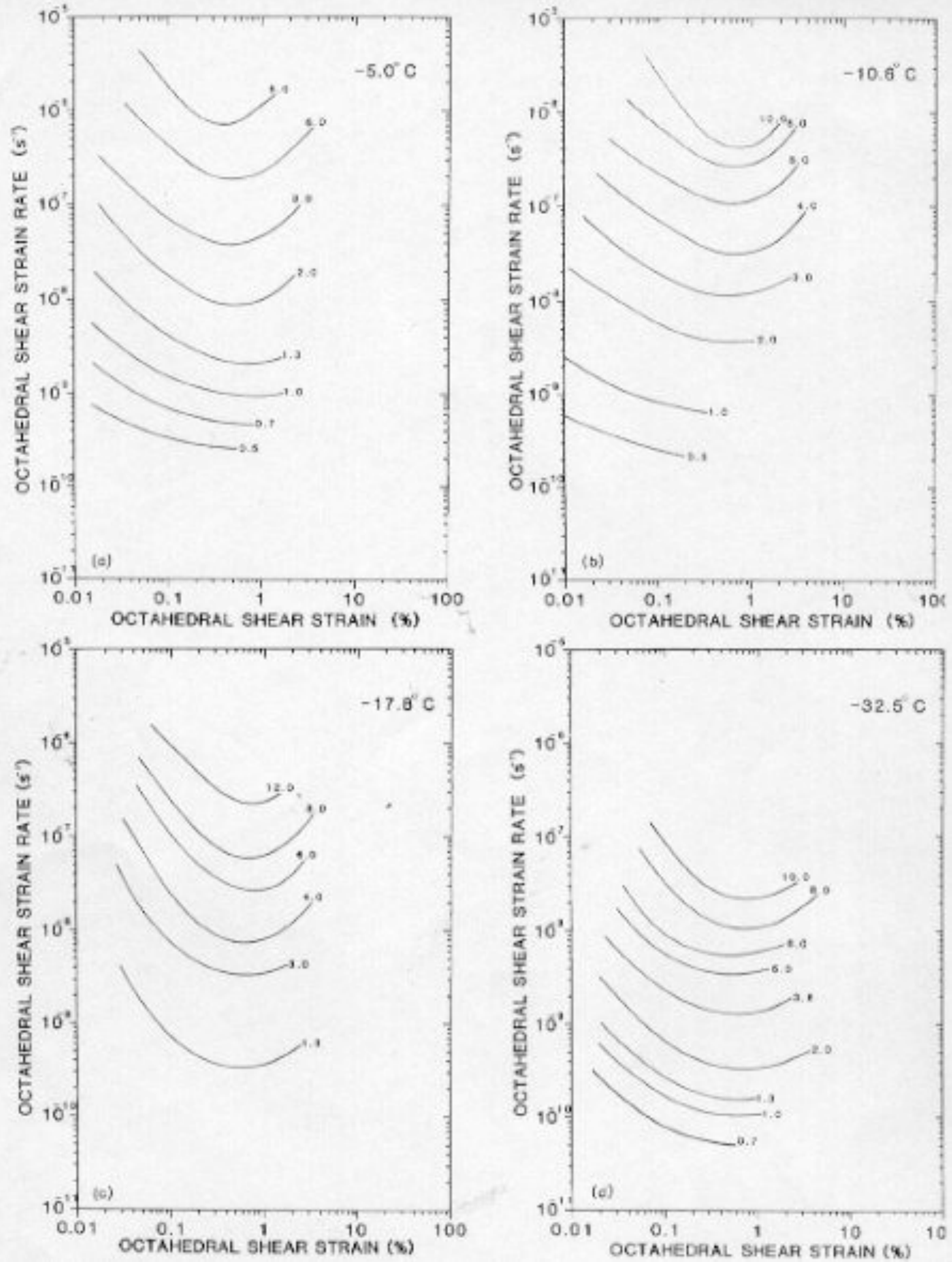




CREEP CURVES OF ISOTROPIC POLYCRYSTALLINE ICE

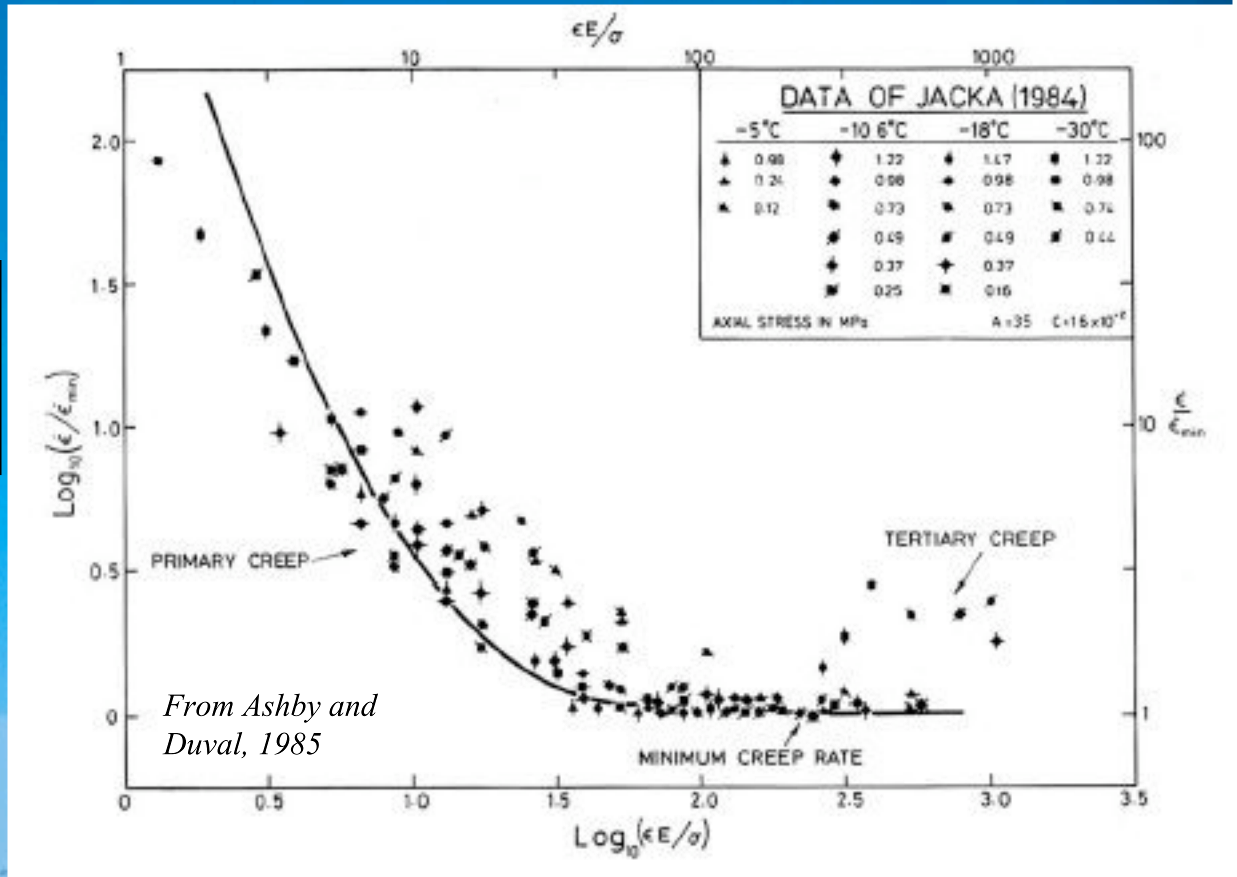
Secondary creep is reached at a strain of about 1%

From Jacka, 1984



Transient creep in polycrystalline ice

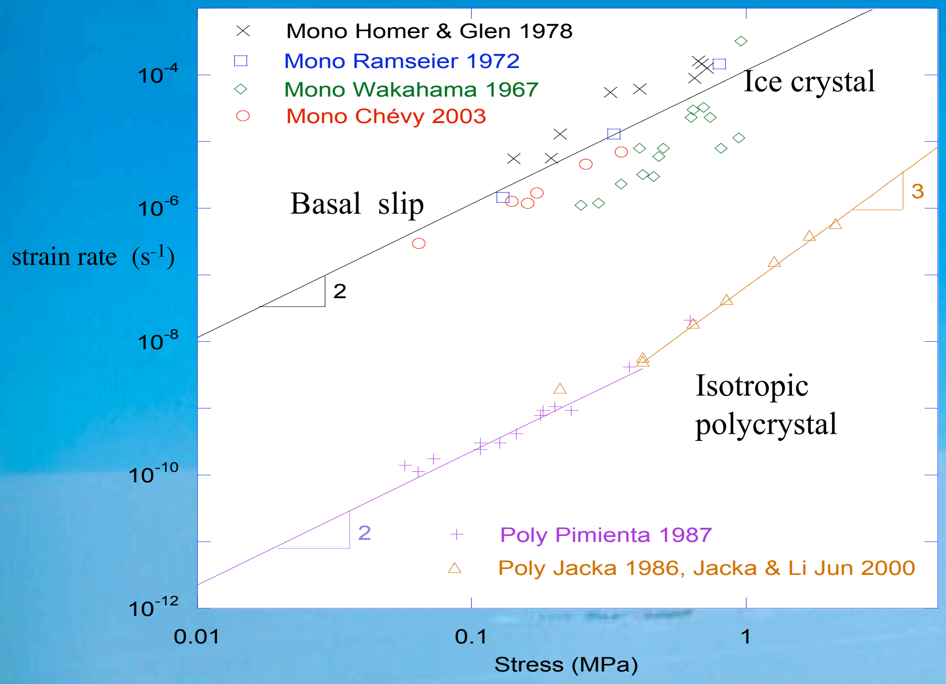
Strain rate is decreasing by a factor higher than 100 during primary creep.

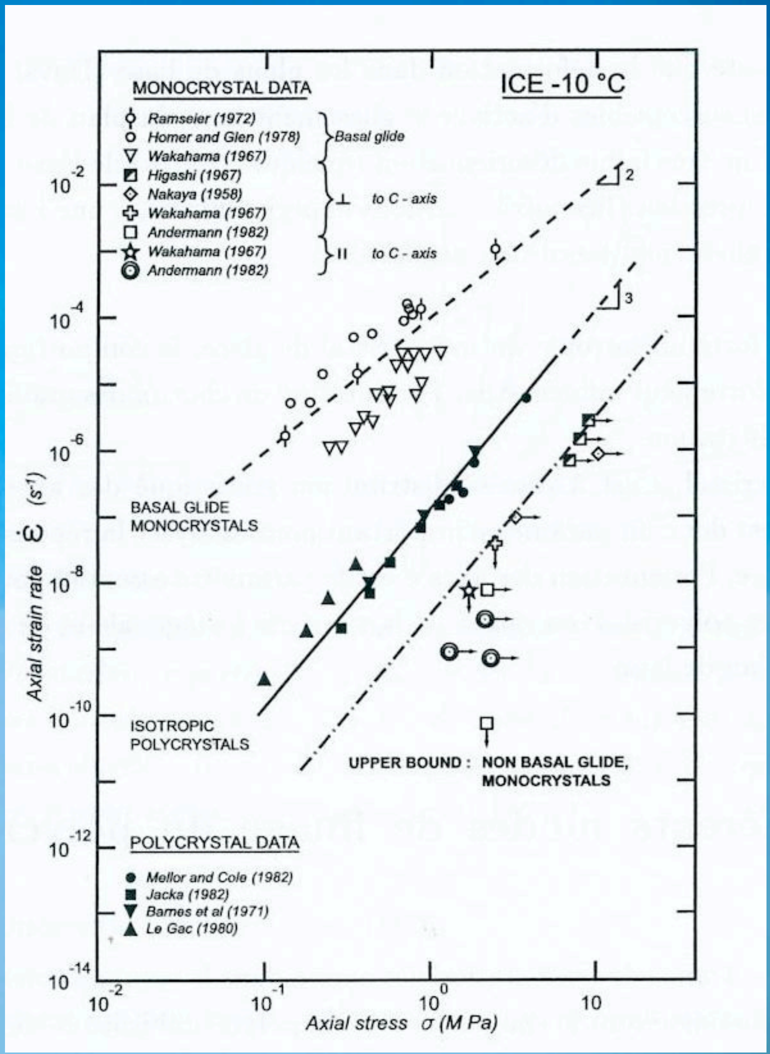


From Ashby and Duval, 1985

The mismatch of slip at grain boundaries induces deformation gradients, long-range internal stresses and kinematics hardening.

Stress-strain rate relationship for the ice crystal and isotropic polycrystalline ice

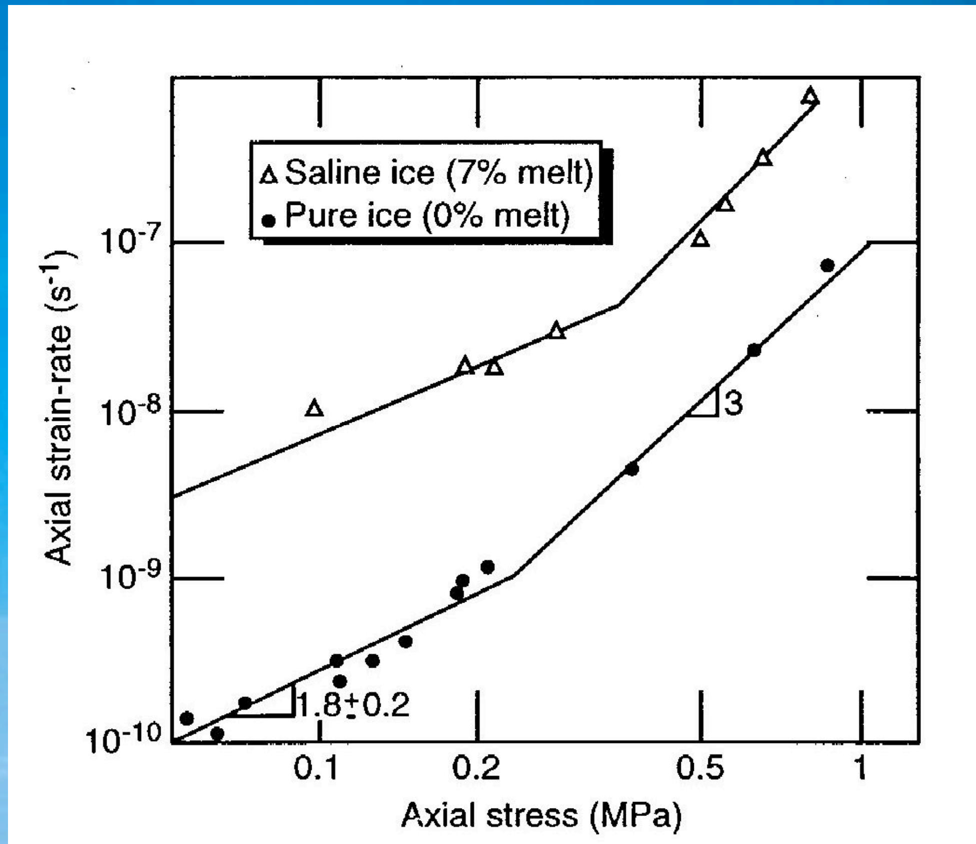




From Duval et al., 1983

Effect of a liquid phase on the creep behavior of isotropic ice

A viscosity lower than 10^{14} Pa.s is expected at the melting point.



From De la Chapelle et al., Geophys. Res. Letters, 1999

RATE-CONTROLLING PROCESSES IN THE CREEP OF POLYCRYSTALLINE ICE

⇒*The deformation is essentially produced by basal slip (two independent slip systems)*

⇒*Other deformation modes: slip on non-basal planes, climb of dislocations, grain boundary sliding (GBS)*

- *at relatively high stresses ($n = 3$), the climb of dislocations should be the controlling process*
- *at low stresses ($n \approx 1.8$), basal slip would be accommodated by GBS and GB migration. A grain size effect is expected.*

→ *Stress and strain fluctuations within grains must be also taken into account*

ICE FLOW LAW FOR ISOTROPIC ICE

$$\dot{\varepsilon}_{i,j} = \frac{B}{2}\tau^2\sigma'_{i,j}$$

with

$$\tau^2 = \frac{1}{2}\sum_{i,j}(\sigma'_{i,j})^2$$

For deviatoric stresses higher than 0.2 MPa. , the stress exponent for secondary creep is about 3

At low stresses, the stress exponent is lower than 2

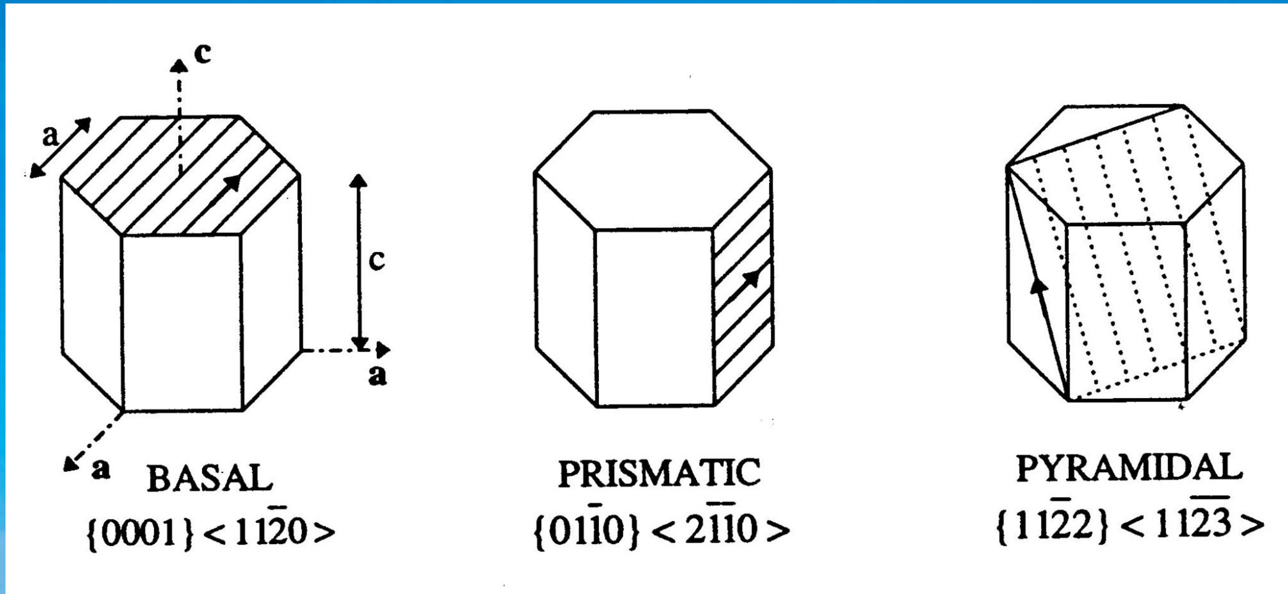
MICRO-MACRO APPROACHES FOR MODELING THE ICE POLYCRYSTAL

Self-consistent models:

- The VPSC tangent model of Lebensohn and Tomé (*Acta mater. 1993*)
- The second-order homogenization procedure of Ponte Castaneda (*J. Mech. and Phys. of Solids, 2002, 2004*)

- Other methods: Finite Element Method or the Fast Fourier Transform *(Lebensohn, 2001)*

Deformation modes of the ice crystal



The shear strain rate on the slip system *s, is:*
$$\dot{\gamma}_s = \dot{\gamma}_o \left| \frac{\boldsymbol{\tau}^s}{\boldsymbol{\tau}_o^s} \right|^{n_s - 1} \left(\frac{\boldsymbol{\tau}^s}{\boldsymbol{\tau}_o^s} \right)$$

τ_0^s is the critical stress for the slip system *s* and n_s is the exponent of the flow law associated with the slip system *s*

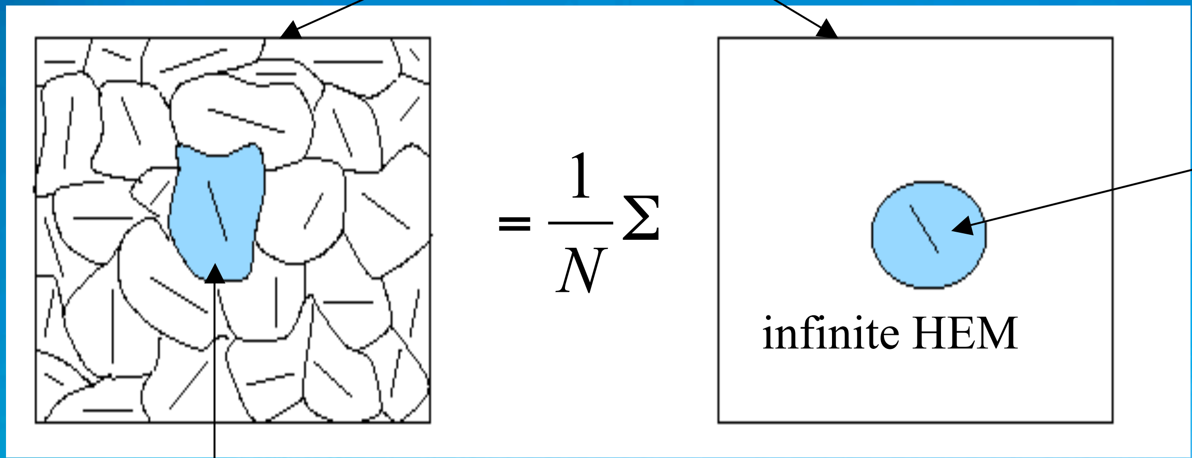
$\boldsymbol{\tau}_s$ is the shear stress on the slip system *s*

The 1 site VPSC tangent model

$\underline{\overline{\sigma}} \quad \dot{\underline{\overline{\varepsilon}}}$

$$\dot{\underline{\varepsilon}}^g - \dot{\underline{\overline{\varepsilon}}} = -\underline{\underline{M}} : \left(\underline{\sigma}^g - \underline{\overline{\sigma}}\right)$$

Interaction equation



$\underline{\sigma}^g \quad \dot{\underline{\varepsilon}}^g$ *uniforms within the inclusion*

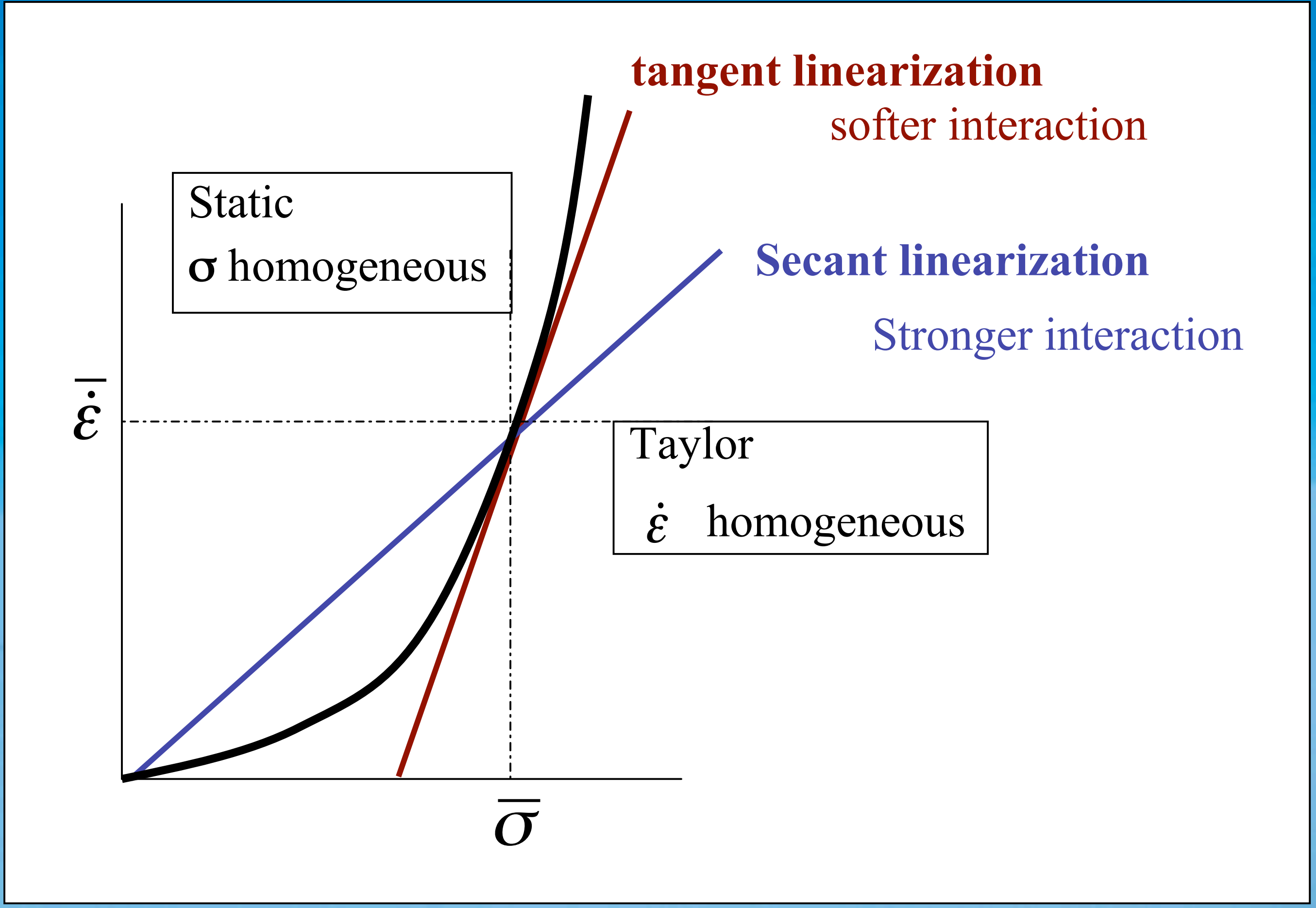
$\underline{\sigma}^g \;,\; \dot{\underline{\varepsilon}}^g$

$$\left\langle \underline{\sigma}^g \right\rangle = \underline{\overline{\sigma}} \quad ; \quad \left\langle \dot{\underline{\varepsilon}}^g \right\rangle = \dot{\underline{\overline{\varepsilon}}}$$

$$\dot{\underline{\varepsilon}}^g = \sum_s r^s \dot{\gamma}_s$$

$$\dot{\underline{\varepsilon}}^g = \sum_s \underline{r}^s \dot{\gamma}_0 \left(\frac{\underline{r}^s : \underline{\sigma}^g}{\tau_0^s}\right)^{n_s}$$

Local linearization of the stress against strain-rate response in the vicinity of overall stress and strain rate



Self-consistent « second-order » approach

Lebensohn et al. Proc. R. Soc Lond, 2004

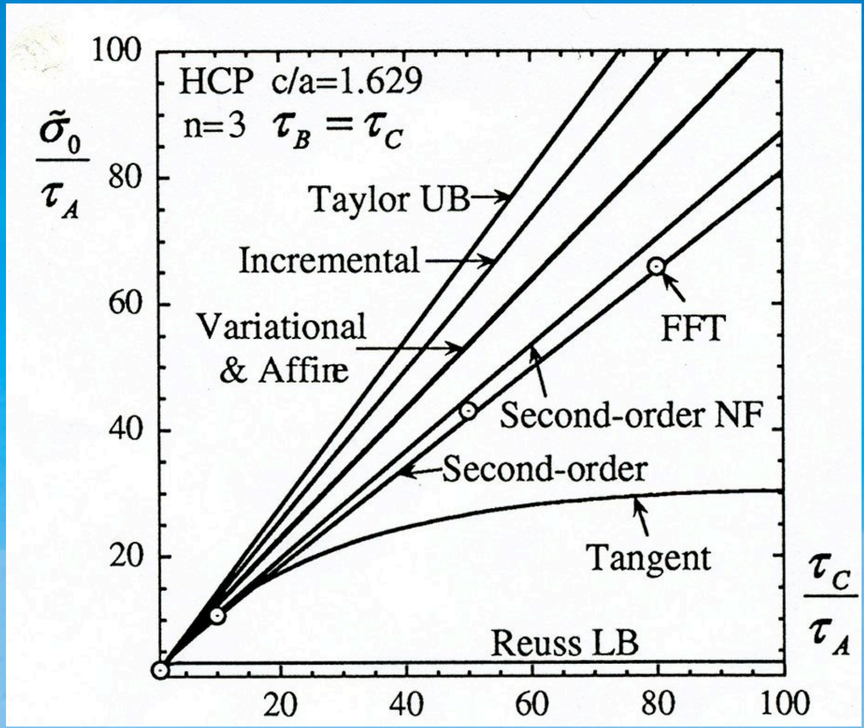
-Incorporates information on the second moments of the stress and strain rate fields.

The second moment of the stress field in a grain r ⟶ $<\boldsymbol{\sigma} \otimes \boldsymbol{\sigma}>_r$ *It gives the grain stress fluctuation*

-Leads to estimates that are exact to second-order in the heterogeneity contrast

Self-consistent estimates for the flow stress for isotropic ice as a function of the grain anisotropy

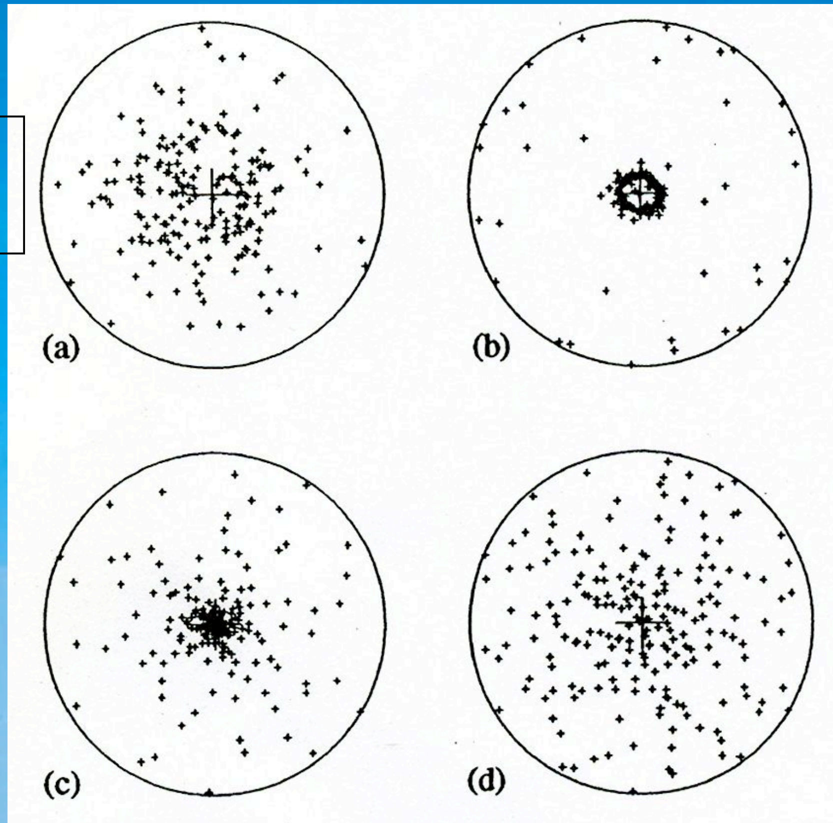
- the second order estimates give the best agreement with the exact solution (FFT)
- the tangent model underestimates the overall behavior for the largest values of the anisotropy factor



From Lebensohn et al., 2004

C-axes textures simulated by the static, Taylor and VPSC models

GRIP ice core at 990m



Static

VPSC

Taylor

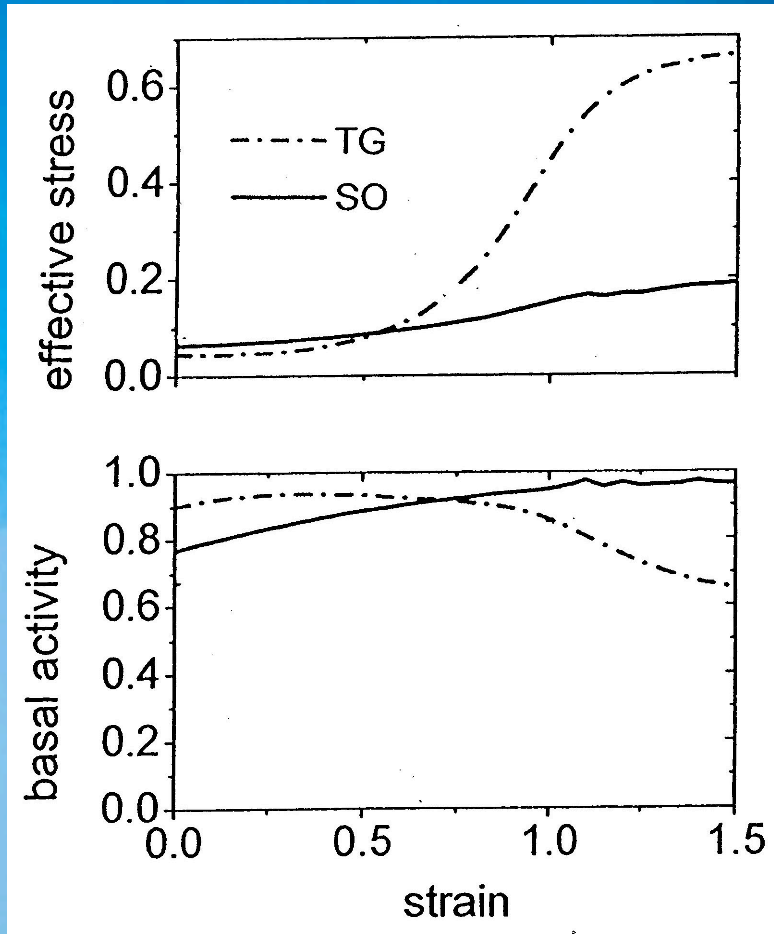
The measured textures lie between the VPSC and Taylor estimates

Simulation of the behaviour of an ice polycrystal under uniaxial compression with the VPSC and second-order models

Calculations were made with n =3,

$\tau_b = 20\ \tau_a$ *and* $\tau_c = 200\ \tau_a$

With the VPSC tangent model, basal slip cannot accommodate the deformation when c-axes become strongly aligned with the compression axis

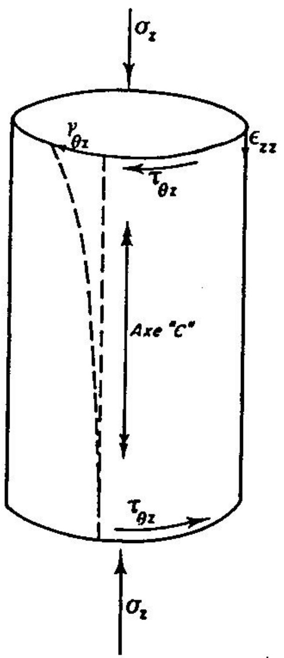


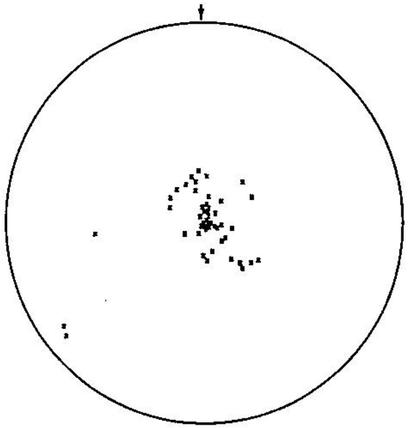
From Lebensohn et al., Phil. Mag., in press

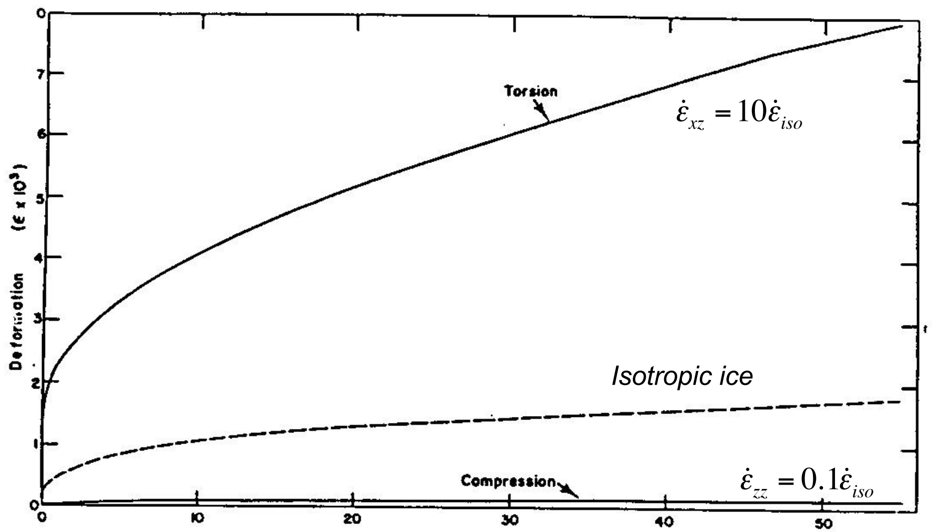
Modelling of the behaviour of isotropic and anisotropic polycrystalline ice

- *stress and strain -rate fluctuations inside grain must be taken into account for anisotropic materials*
- *the second-order model leads to estimates that are exact to second-order in the heterogeneity contrast; it predicts a good evolution of textures and gives the average field fluctuations in grains*

Viscosity of anisotropic ice: experimental measurements







The viscosity in compression is about 100 times higher than the viscosity in torsion and 10 times higher than that for isotropic ice

Anisotropy of ice and the flow of ice sheets

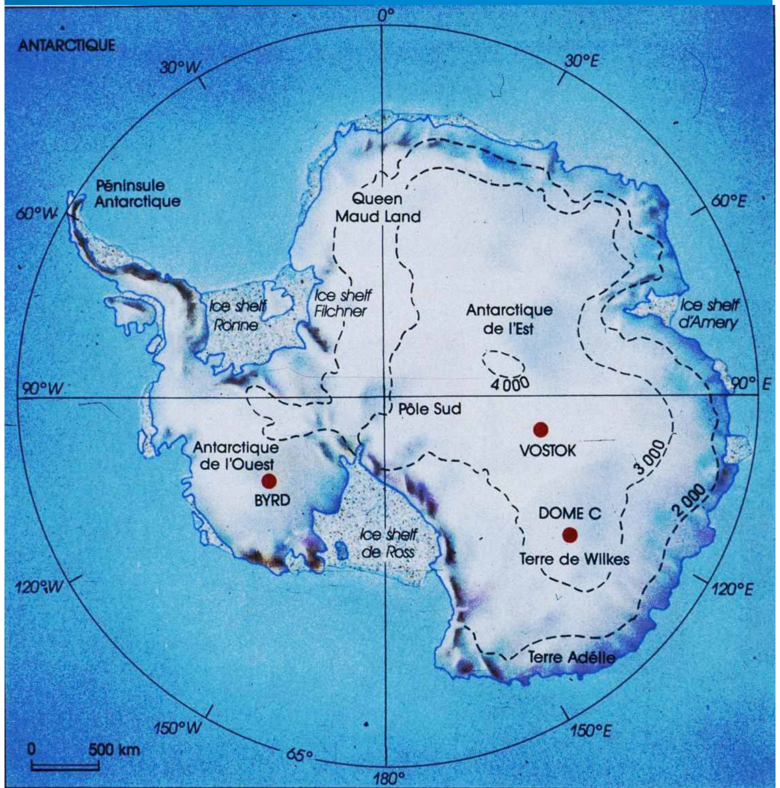
The ice sheet flow model:

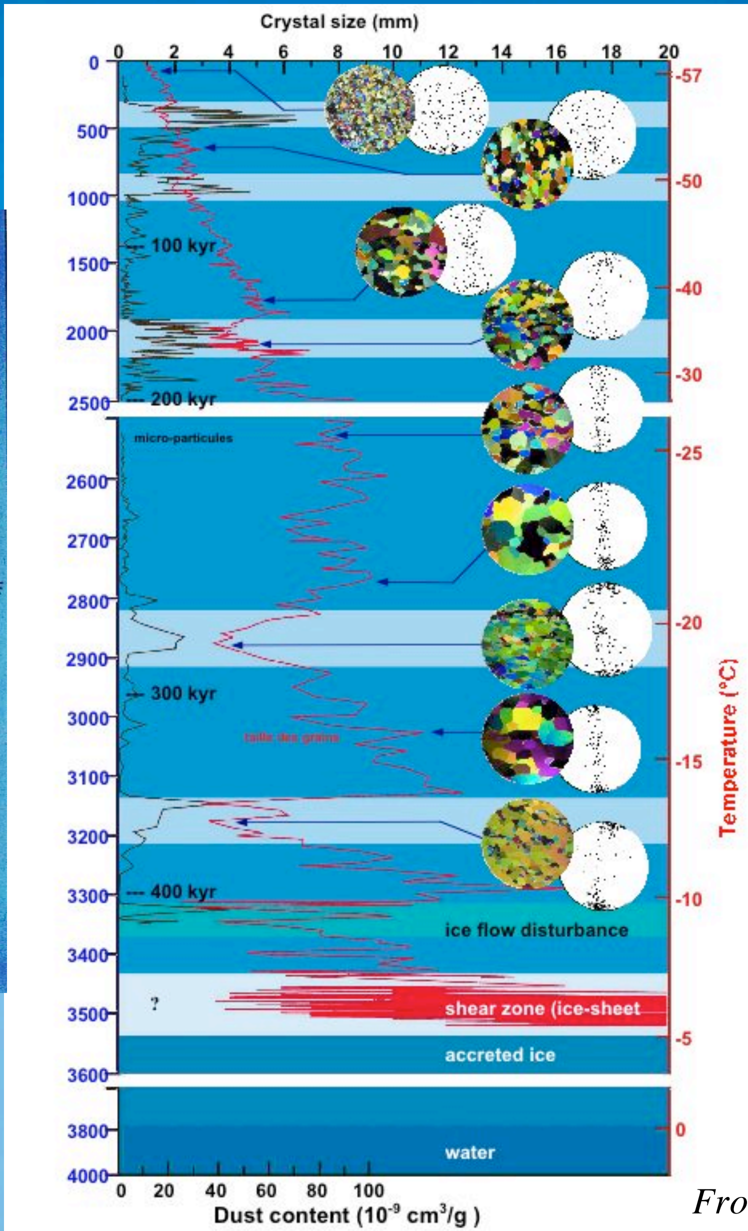
- *equations of continuity*
- *conservation of momentum*
- *ice flow law with texture evolution*
- *free surface equation*
- *heat equation*
- *conditions at the sub-glacial interface*

A 3600 m ice core at Vostok

- *In East Antarctica the age of ice can be more than 700,000 years at the depth of 3000 m*

Structure of ice along the 3600 m Vostok ice core





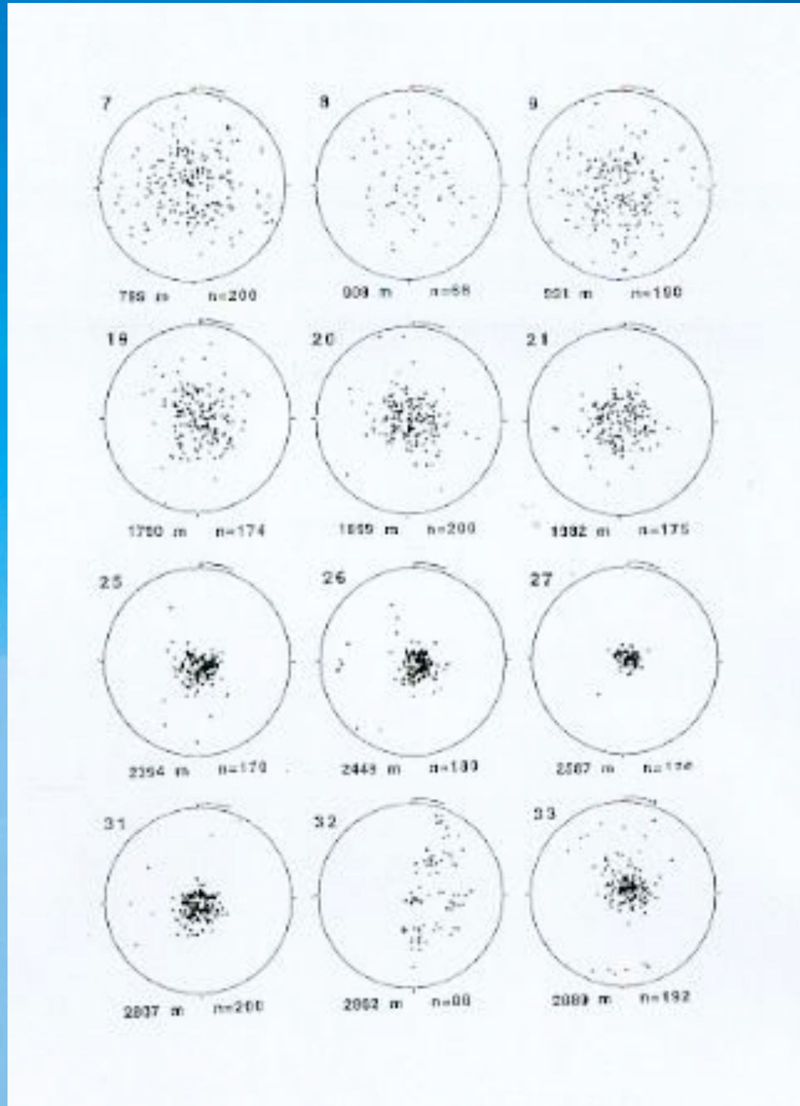
- Normal grain growth occurs in the first 1000m and continuous recrystallization below

- C-axes textures are compatible with extension along an horizontal direction; but, simple shear is clearly dominant at large depth

From Lipenkov, 1998

C-AXES TEXTURE ALONG THE 3,000m GRIP ICE CORE (GROENLAND)

The progressive rotation of c-axes towards the vertical direction is due to uniaxial compression along the vertical direction. Horizontal simple shear can be also invoked below 2,000m.



From Thorsteinsson et al., J. of Geoph. Res., 1997

Anisotropy of ice and the flow of ice sheets

The flow law

→The standard flow law is often used in ice flow models:

$$\dot{\varepsilon}_{i,j} = EA_o \exp(-Q/RT)\tau_{eff}^{n-1}\tau_{i,j}$$

E is called the enhancement factor introduced to take into account the effect of textures.

→this flow law can be only used when a single component of the stress tensor dominates the flow of the ice sheet.

→for other cases, the components of strain rates are not proportional to the corresponding components of the stress tensor.

The continuum model

- *The texture is described by an orientation distribution function (ODF);*

$f(\theta, \phi)$ gives the density of c axes with the orientation θ, ϕ with

$$\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2} f(\theta,\varphi)\sin\theta d\theta d\phi = 1$$

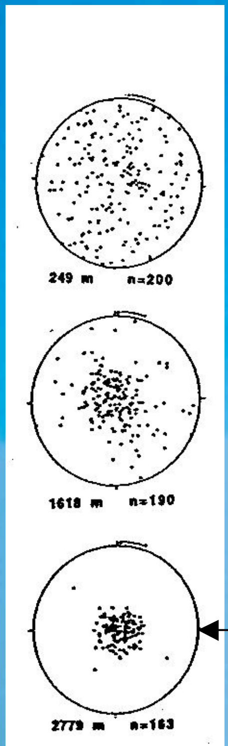
-the stress is assumed uniform within the polycrystal (Sach's assumption)

$$\Rightarrow \left\langle \underline{\dot{\varepsilon}}^g \right\rangle = \underline{\bar{\dot{\varepsilon}}}$$

$$\underline{\dot{\varepsilon}}^g = \sum_s \underline{r}^s \dot{\gamma}_0 \left(\frac{\underline{r}^s : \underline{\sigma}^g}{\tau_0^s}\right)^{n_s}$$

The role of the anisotropy of ice in flow near an ice divide

Isotropic ice near the surface and strong vertical c-axis texture in depth



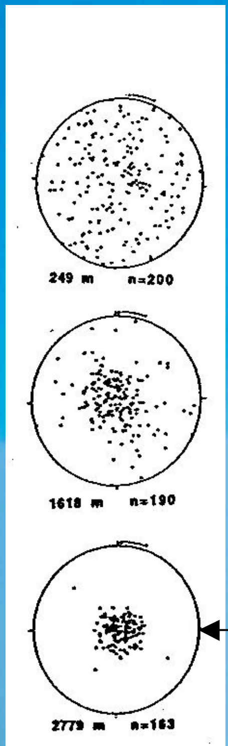
C-axes texture along the GRIP ice core

Assuming a non-linear law with n =3

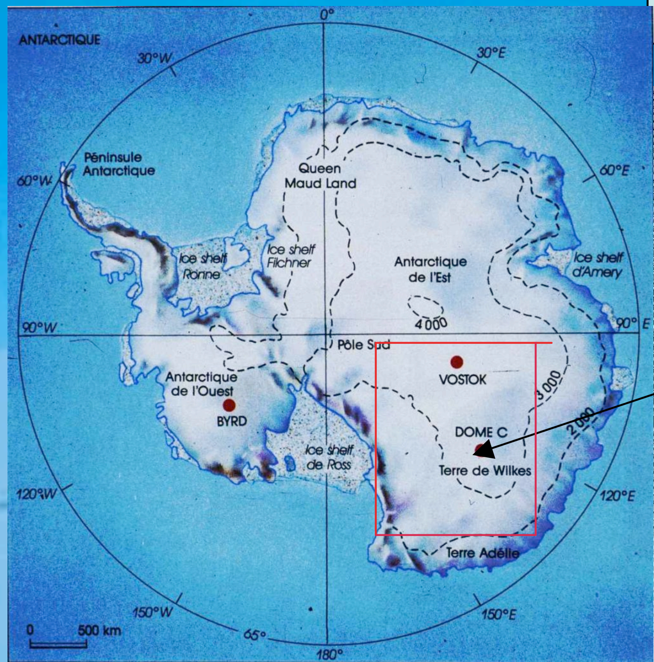
-The viscosity in compression along the symmetry axis of this texture (the vertical direction) is 4 times higher than that for isotropic ice
- the viscosity for horizontal shear is 10 times lower than that for isotropic ice

The role of the anisotropy of ice in flow near an ice divide

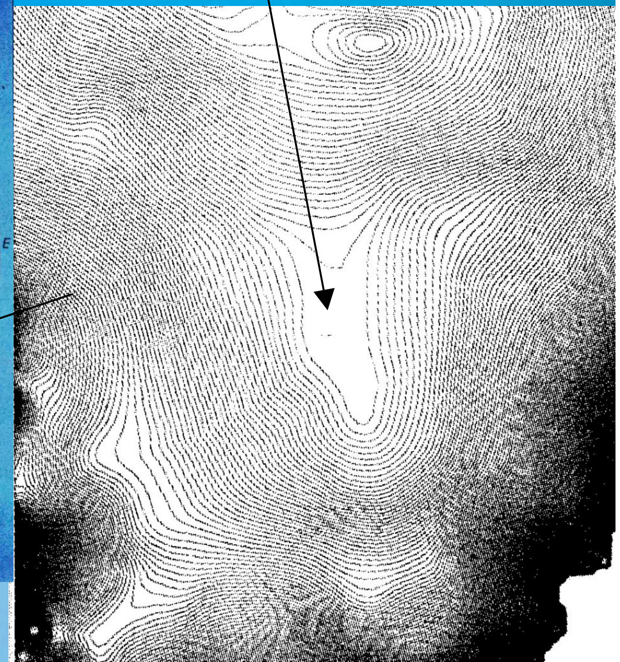
Isotropic ice near the surface and strong vertical c-axis texture in depth



C-axes texture along the GRIP ice core



Ice divide near Concordia (East Antarctica)



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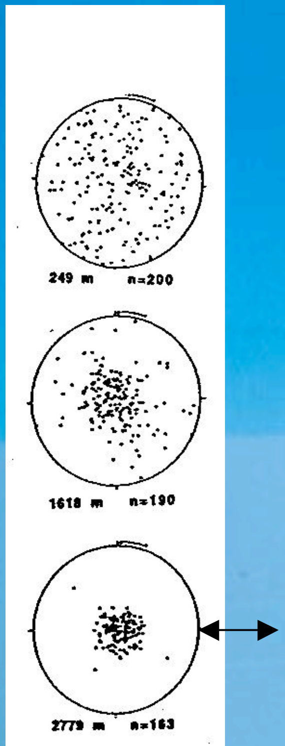
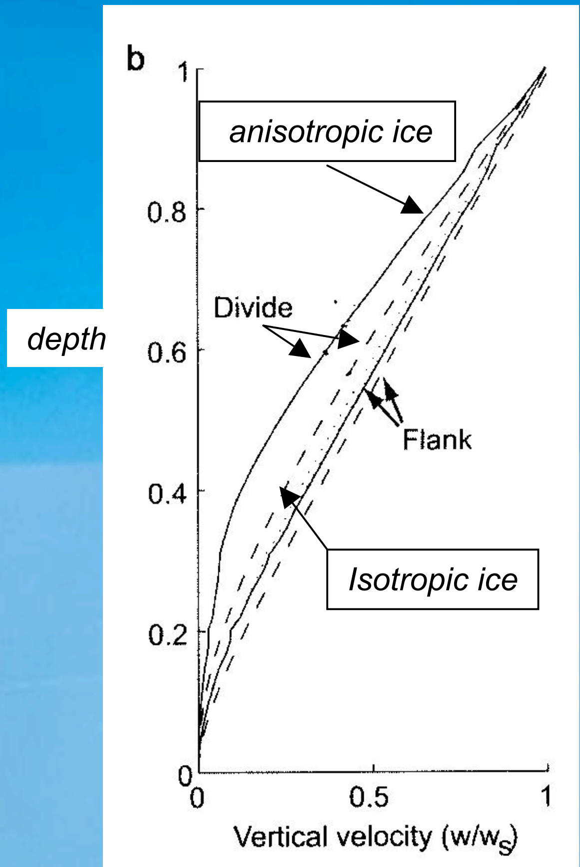


C-axes texture along the GRIP ice core



from Pettit et al., J. of Glaciology, 2007

Textures were imposed

-the vertical velocity at the divide obtained with anisotropic ice shows higher curvature
- this model gives higher velocities than models with isotropic ice

The datation of deep ice cores deduced from ice sheet flow modelling can be improved by taking into account the anisotropy of ice

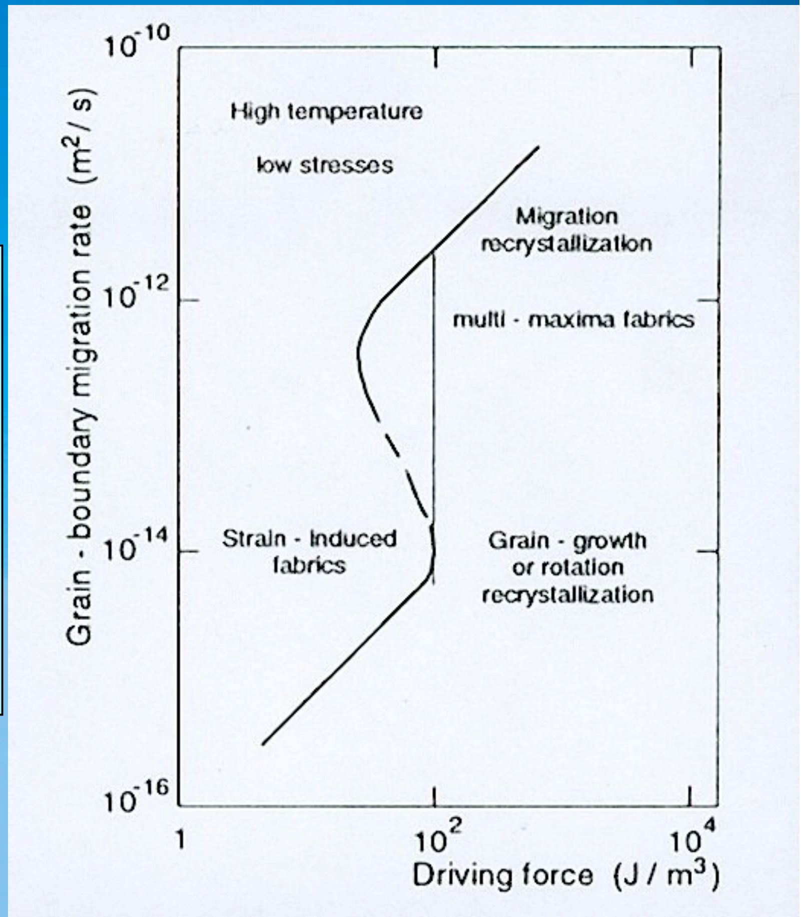
DATATION OF DEEP ICE CORES BY ICE FLOW MODELING

⇒ the age of an ice layer is computed with an estimation of the variation with time of the accumulation rate and the thinning function (*the ratio of the thickness of a layer to its initial thickness*).

The thinning function is computed by a 2 or 3D flow model. This function is depending on the ice viscosity which can be directional when ice is anisotropic.

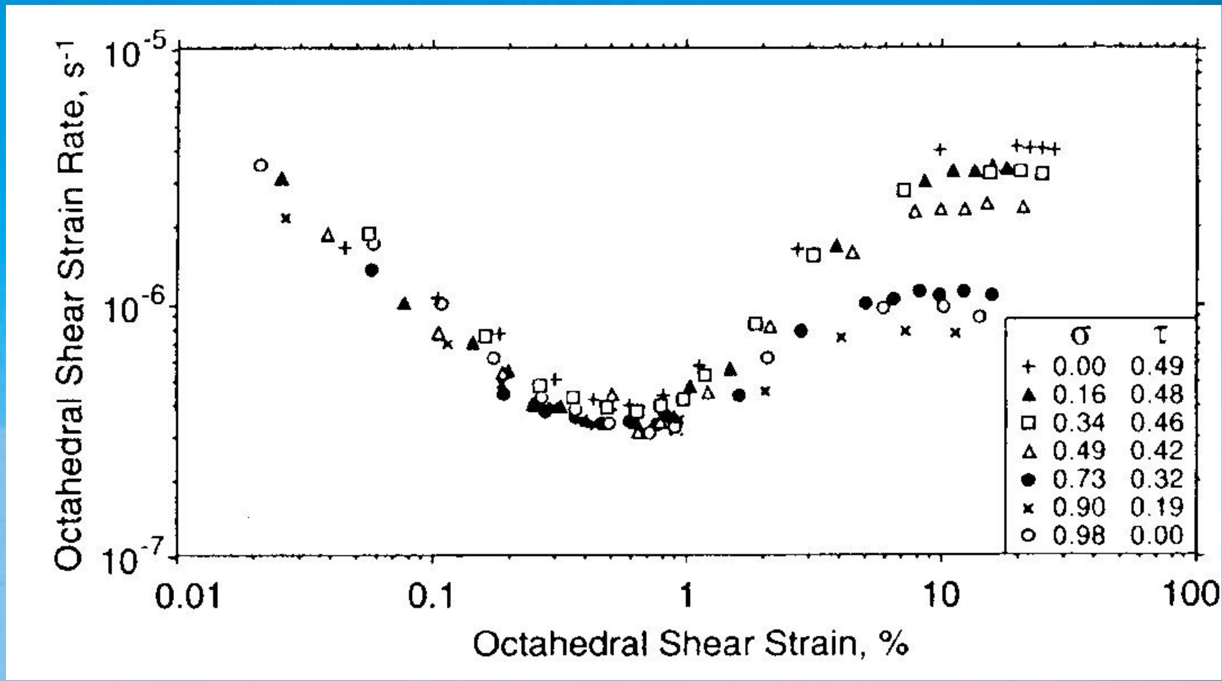
The flow law of anisotropic ice and dynamic recrystallization

-when migration recrystallization occurs, textures are induced by recrystallization and are stress-controlled. Ice must be considered as isotropic.
- textures are strain-induced when rotation recrystallization occurs; ice is anisotropic



TERTIARY CREEP RATE AS A FUNCTION OF THE STRESS STATE

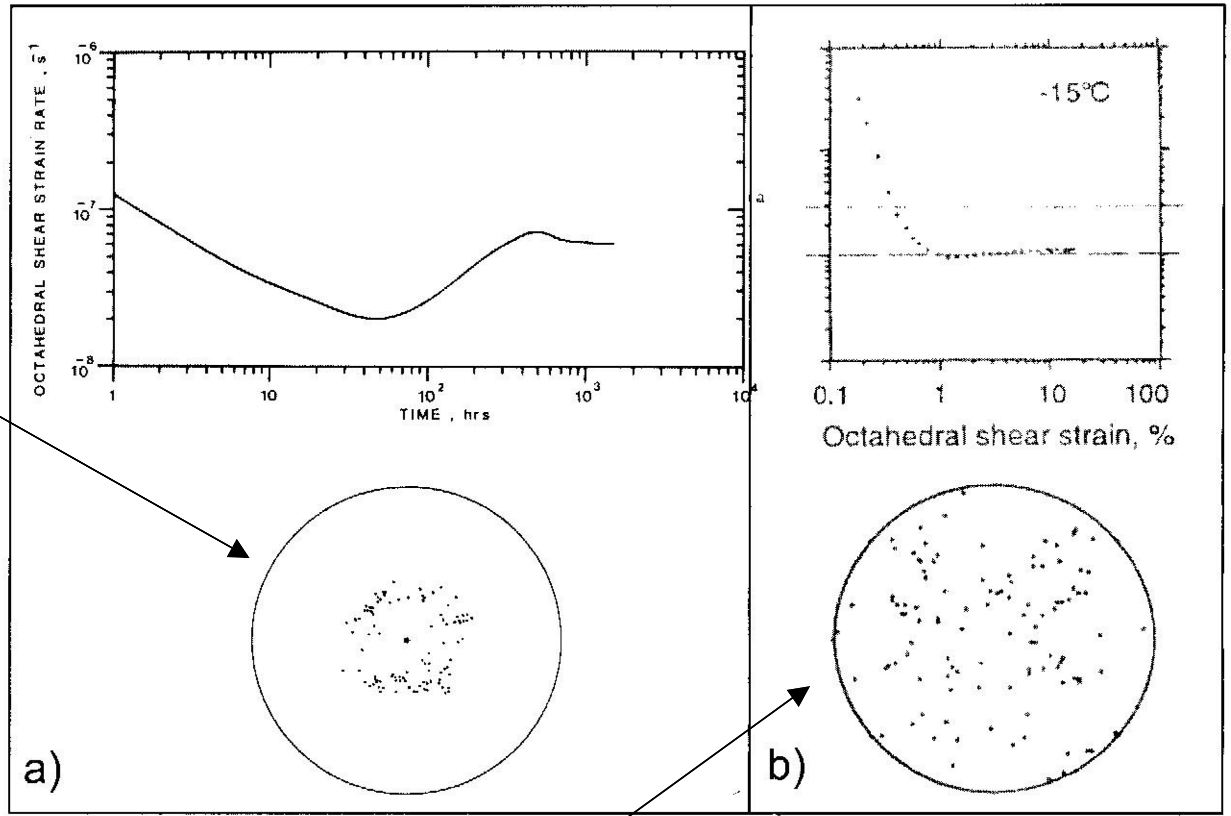
The tertiary creep rate during dynamic recrystallization can be more than 10 times the secondary creep rate in simple shear



From Li Jun et al., 1996

TEXTURES DEVELOPMENT DURING CREEP

Recrystallization textures form at a relatively low strain (≤ 15%)



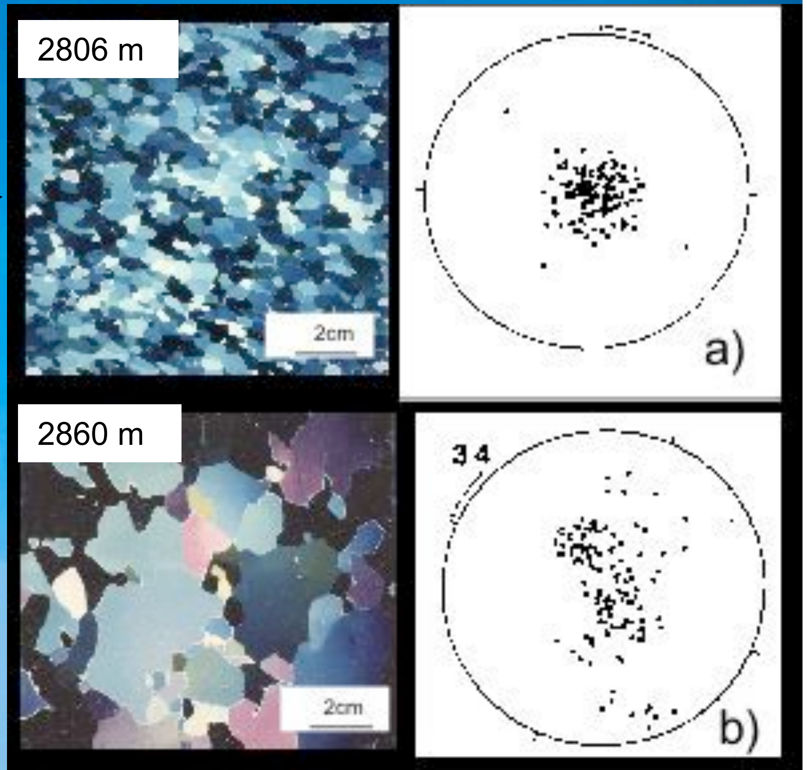
Strain less 20% is not enough high to form significant deformation textures

From Jacka and Li Jun, 2000

DYNAMIC RECRYSTALLIZATION IN GREENLAND ICE

Rotation recrystallization and deformation textures

Migration recrystallization and recrystallization textures



From Thorsteinsson, et al., 1997

These two recrystallization regimes are observed at about the same depth along the 3,100 m GRIP ice core; variations of the impurity content would be at the origin of these two recrystallization processes

Conclusions

- The viscoplastic anisotropy of the ice crystal is exceptionally high
- In glaciers and ice sheets, deformation is essentially produced by dislocation slip
- Depending on textures and dynamic recrystallization processes , polar ice is can be a strongly anisotropic material
- For ice sheet flow modeling, a flow law derived from micro-macro approaches, which take into account stress and strain rate heterogeneities within grains, can be used with new computational possibilities

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