Seismic Tomography

Past, Present, Future

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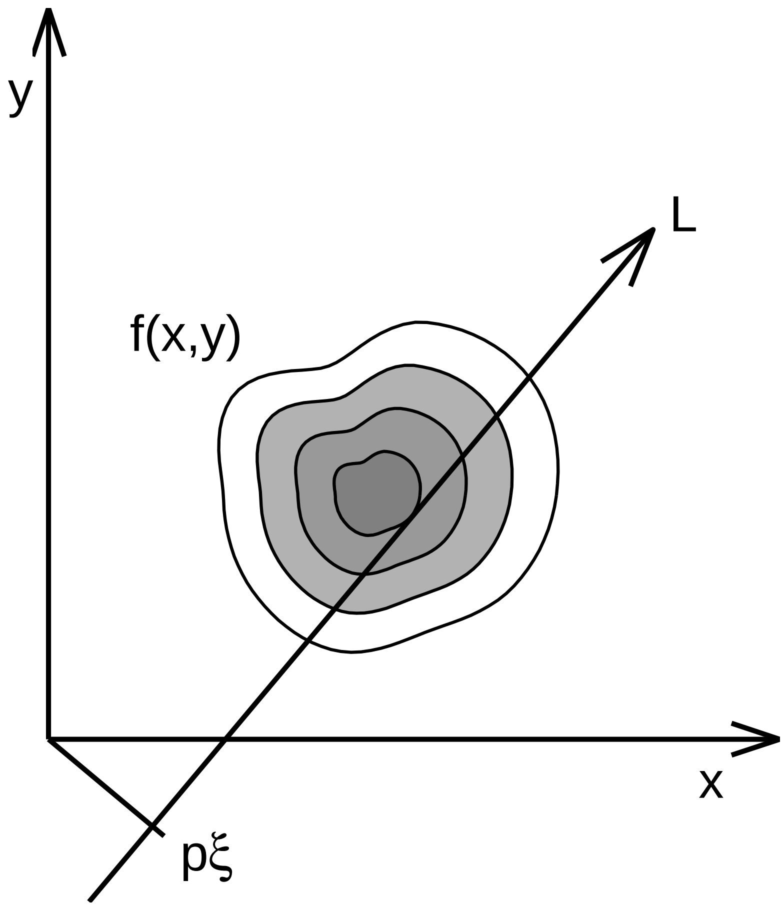
In memoriam

Tony Dahlen

(1942–2007)

Historical beginnings

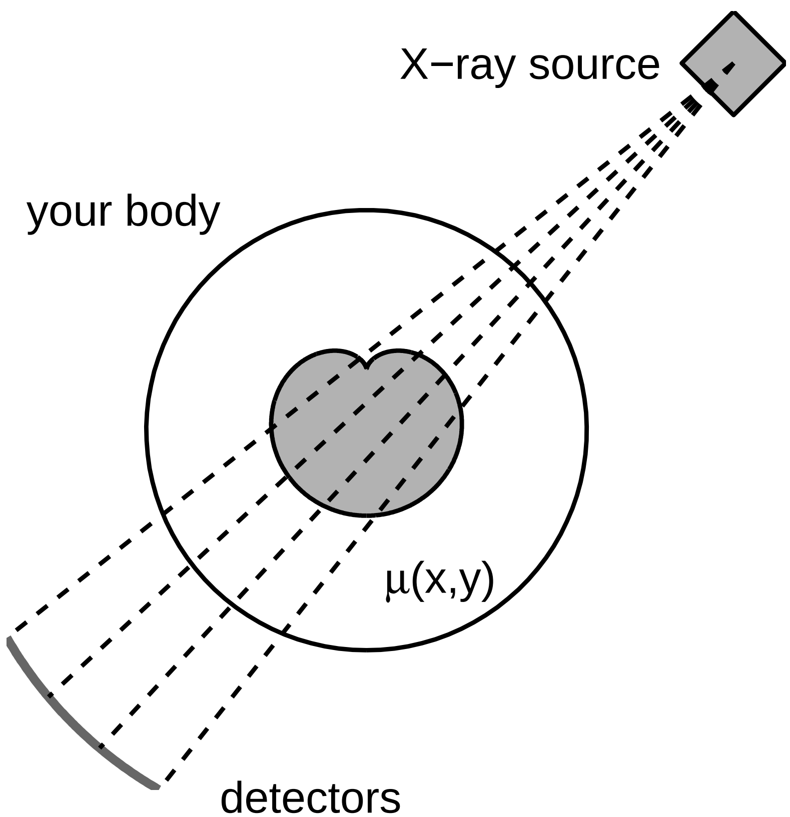


Inverting the Radon transform

$$\mathcal{R}[f](p, \boldsymbol{\xi}) = \int_L f(x, y)\, dl. \qquad (1)$$

Reconstruct the function from its projections: given $\mathcal{R}[f](p, \boldsymbol{\xi})$, find $f(x, y)$.

Radon [1917] solved to this problem, giving an expression for $\mathcal{R}^{-1}$ for straight “ray paths”.



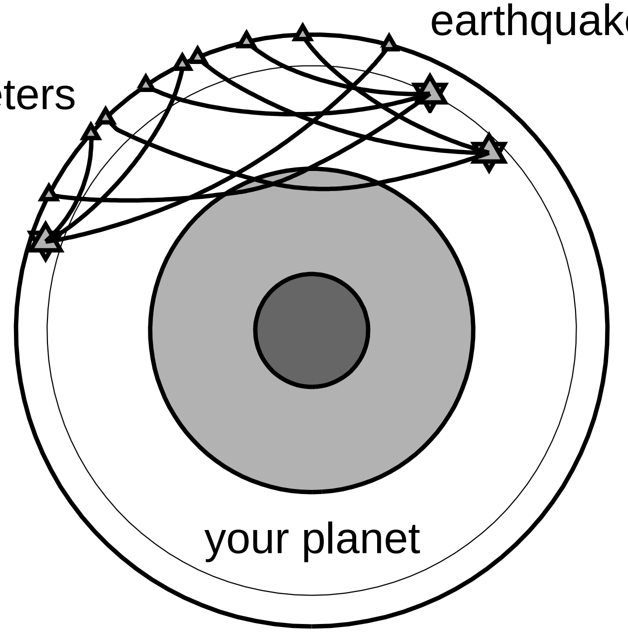
X-ray absorption & scattering

Tissues and bones have different absorption and scattering coefficients $\mu(x, y)$.

Recorded intensity goes as

$$I = I_0 \exp\left[\int_{\text{ray}} -\mu(x, y)\, dl\right]. \quad (2)$$

The exponential is linearized. Sources and detectors rotate to achieve perfect "coverage".



Travel-time tomography

The Earth has a heterogeneous wave-speed structure $c(\mathbf{r}) = c_0(\mathbf{r}) + \delta c(\mathbf{r})$.

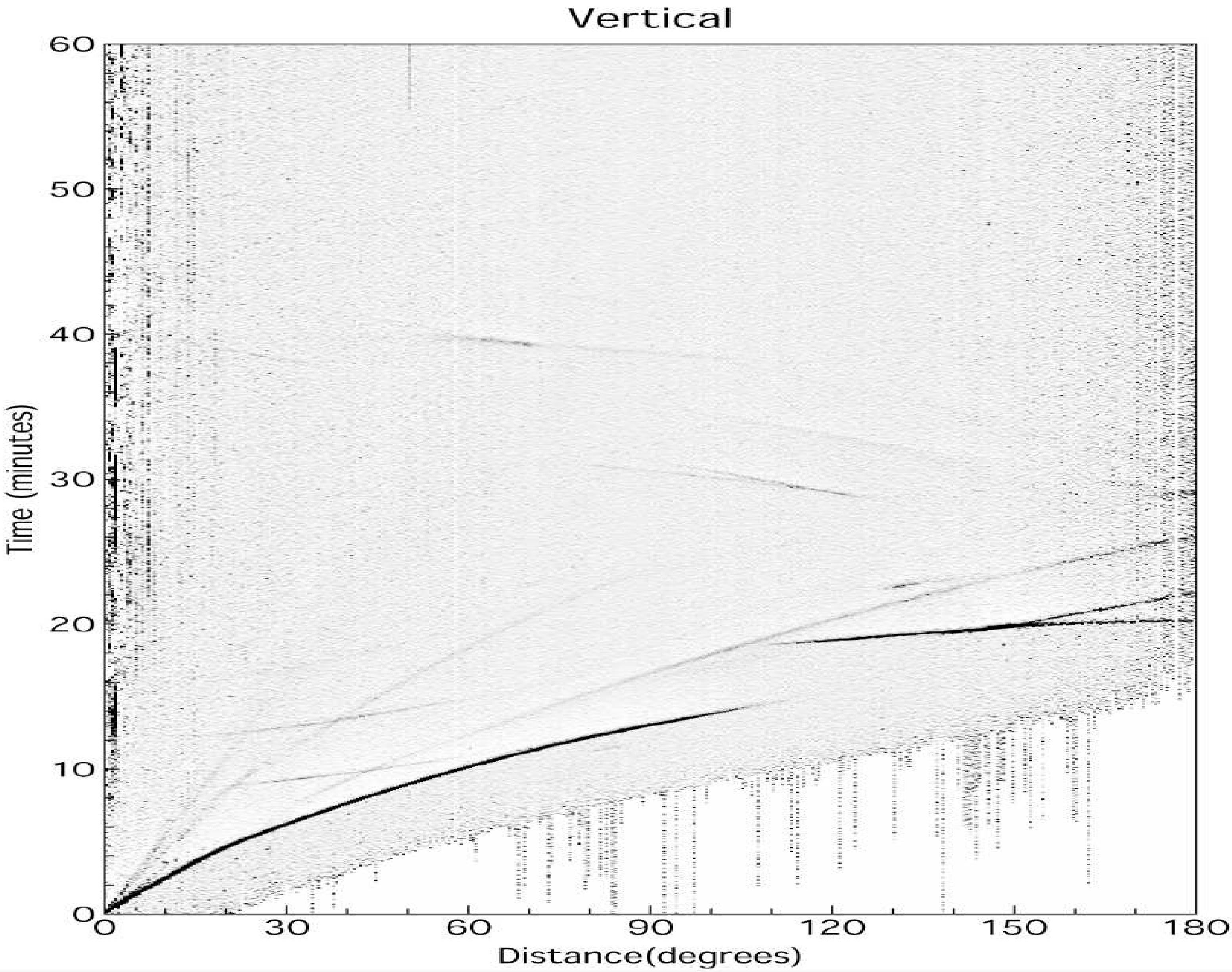
Ray-theoretical travel-time anomalies are

$$\delta t \approx \int_{\text{ray}} \delta c^{-1}\, dl \approx -\int_{\text{ray}} \frac{\delta c}{c_0^2}\, dl. \qquad (3)$$

Fermat's principle allows ray to be calculated in the reference model $c_0(\mathbf{r})$.

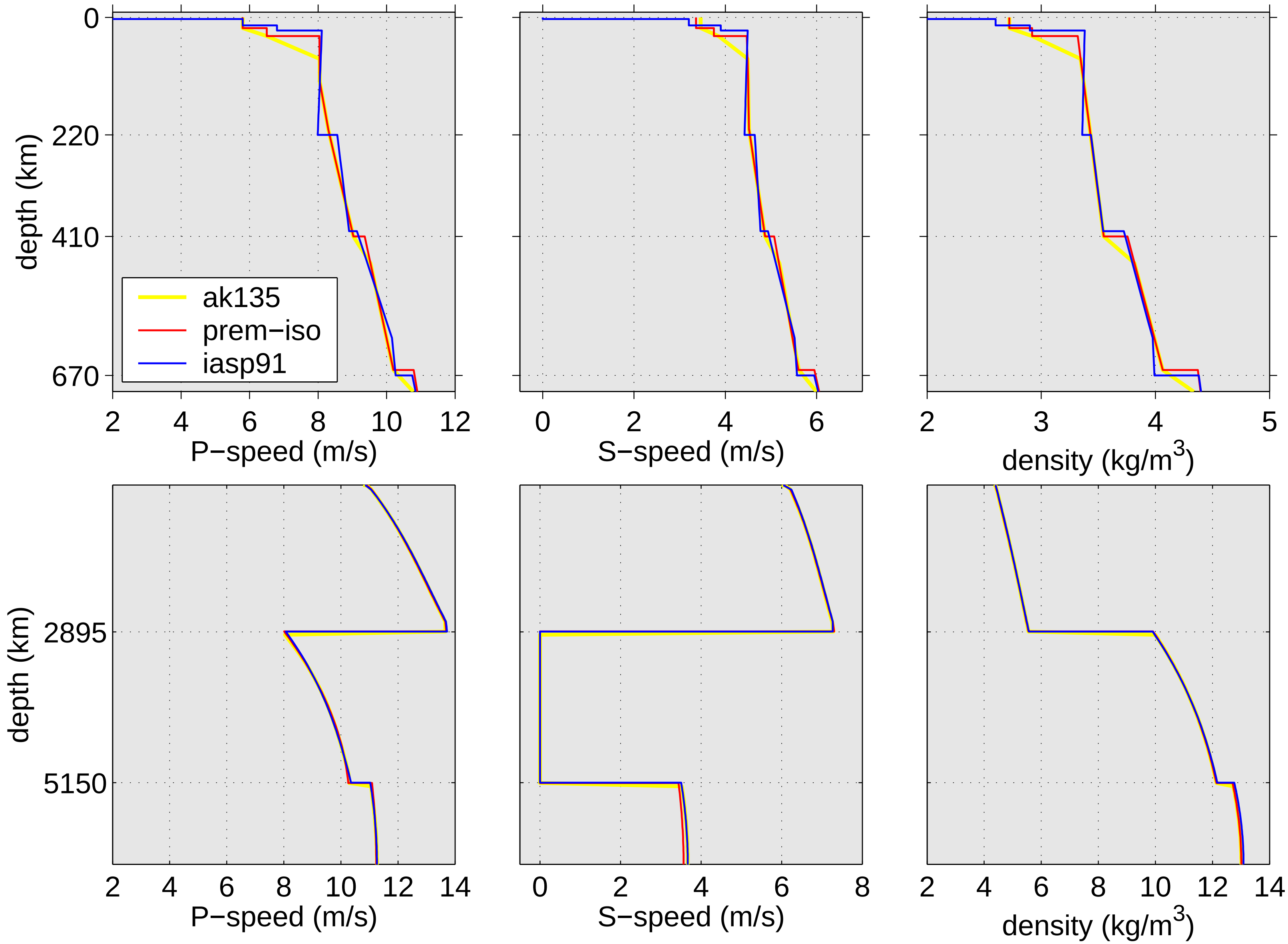
Usually, not exclusively, $c_0(\mathbf{r}) = c_0(r)$.

One-dimensional reference Earth models – 1



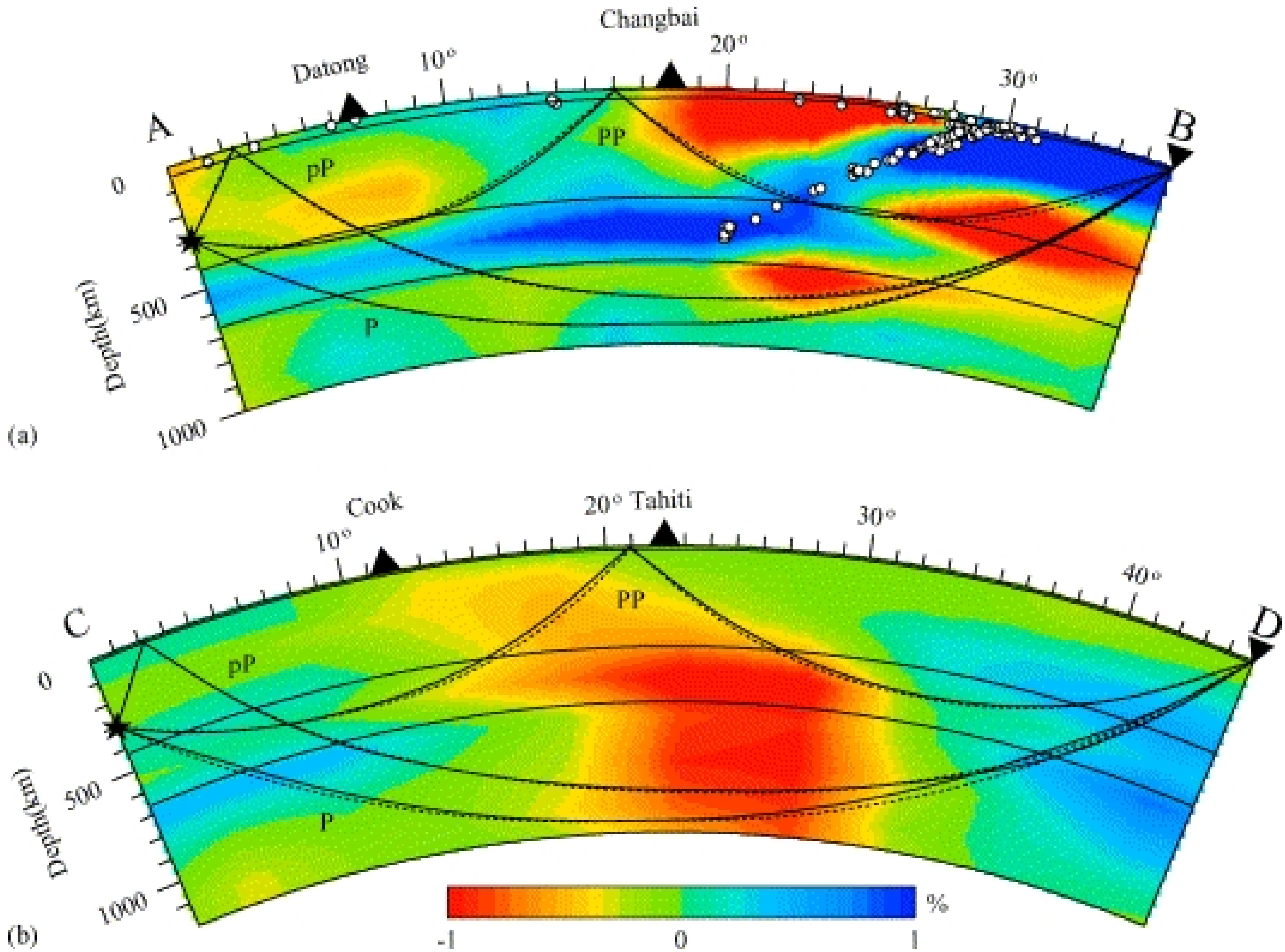
[*Astiz et al.*, 1996]

One-dimensional reference Earth models – 2



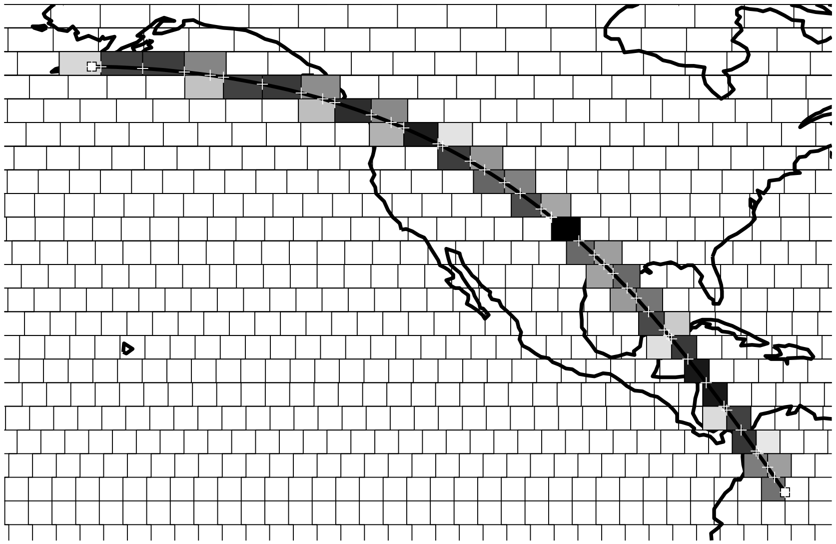
[*Dziewoński and Anderson*, 1981; *Kennett and Engdahl*, 1991]

Fermat's principle



[*Zhao and Lei*, 2004]

Discretization and parameterization



For a set of seismic rays $i = 1 \rightarrow M$, calculate the length spent in each of the $j = 1 \rightarrow N$ grid boxes in which it accumulates a proportional fraction of the total travel-time anomaly δt, discretizing (3). Let's do this for *slowness* here.

$$\delta t_i = L_{ij} \delta s_j \quad \text{or} \quad \boldsymbol{\delta}\mathbf{t} = \mathbf{L} \cdot \boldsymbol{\delta}\mathbf{s} \quad \text{or indeed} \quad \mathbf{G} \cdot \mathbf{m} = \mathbf{d}. \tag{4}$$

$$\text{M travel-time anomalies} \quad \begin{bmatrix} \vdots \\ \delta t_i \\ \vdots \end{bmatrix} = \begin{bmatrix} & \vdots & \\ \ldots & L_{ij} & \ldots \\ & \vdots & \end{bmatrix} \times \begin{bmatrix} \vdots \\ \delta s_j \\ \vdots \end{bmatrix} \quad \text{N slowness perturbations} \tag{5}$$

M×N **sensitivity matrix**

Solving the inverse problem

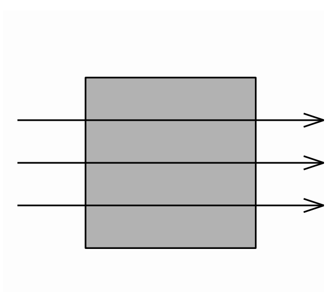
We have: $\mathbf{G} \cdot \mathbf{m} = \mathbf{d}$, which is **linear**.

You think: $\mathbf{m} = \mathbf{G}^{-1} \cdot \mathbf{d}$, but we **can't invert** a non-square $M \times N$ matrix.

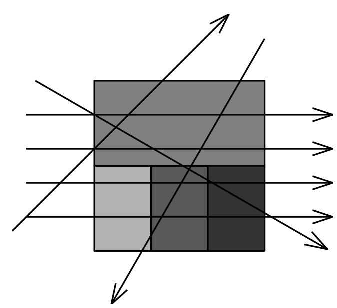
You think: $\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G}$ is square, let's solve $\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G} \cdot \mathbf{m} = \mathbf{G}^{\mathrm{T}} \cdot \mathbf{d}$.

You try: $\mathbf{m} = (\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^{\mathrm{T}} \cdot \mathbf{d}$.

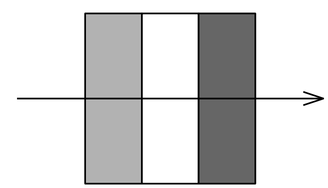
Alas! $\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G}$ may be singular, ill-conditioned, under/over-determined, have (near-)zero eigenvalues, and thus be **not-invertible**. We need more tricks.



over-determined, $M>N$



mixed-determined

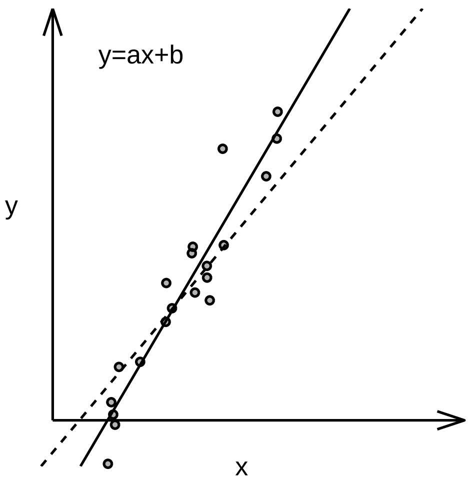


under-determined, $M<N$

[*Menke*, 1989; *Aster et al.*, 2005]

Regularization and a priori information

Over-determined (more data than unknowns):



Define a *penalty fuction* Φ on the *error* $\mathbf{e}$,

and minimize, by least-squares:

$$\Phi = [\mathbf{G} \cdot \mathbf{m} - \mathbf{d}]^2 = \mathbf{e}^{\mathrm{T}} \cdot \mathbf{e}. \quad (6)$$

This is a minimization in the *data space*.

Under-determined (more unknowns than data):

Add equations that minimize some norm in the *model space*:

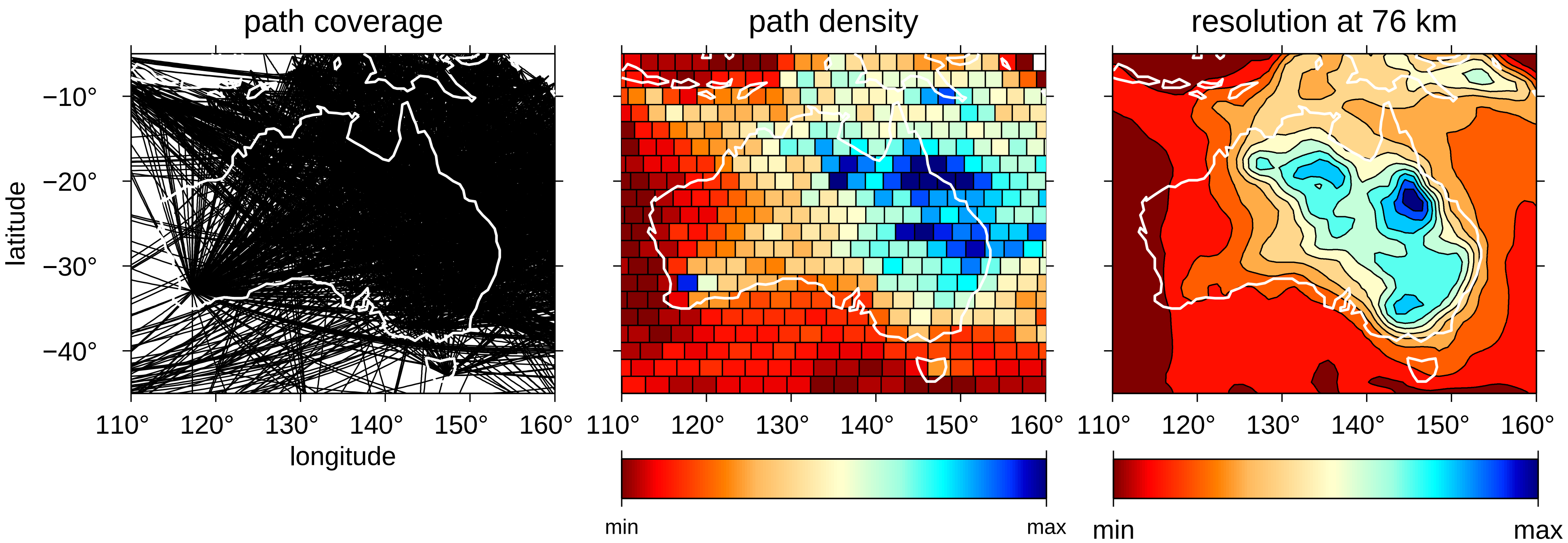
$$\Phi = \mathbf{e}^{\mathrm{T}} \cdot \mathbf{e} + \mathbf{m}^{\mathrm{T}} \cdot \left(\mathbf{A}^{\mathrm{T}} \cdot \mathbf{A}\right) \cdot \mathbf{m}. \quad (7)$$

If $\mathbf{A} = \mathbf{I}$ the identity matrix $\rightarrow$ minimum model norm: **norm damping**.

If $\mathbf{A} = \mathbf{D}$ a difference matrix $\rightarrow$ minimum-roughness: **smoothing**.

Sensitivity: Coverage and resolution

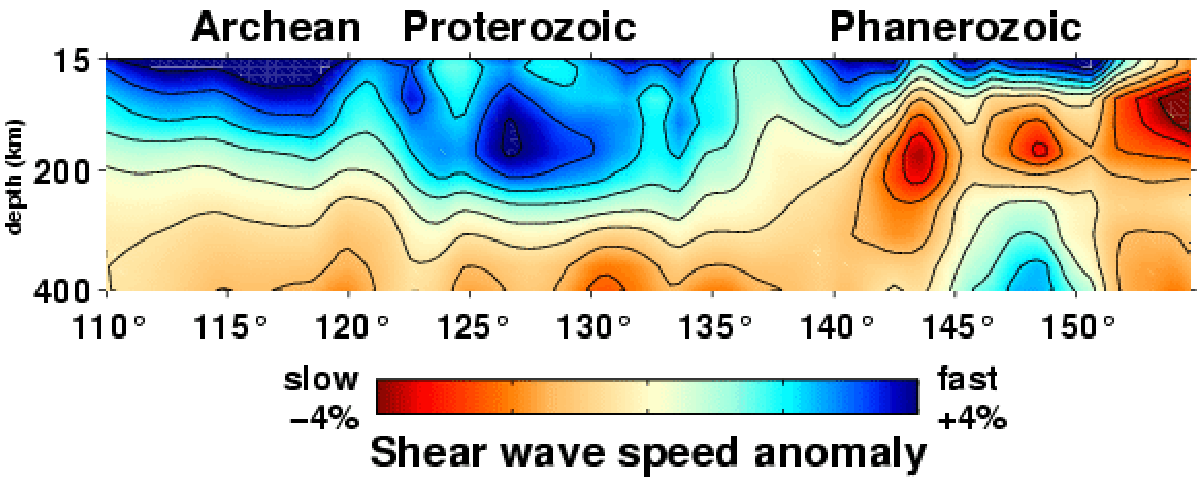
Linearization and discretization produce a dependence of the model $\mathbf{m}$ to the data $\mathbf{d}$ that can be intepreted easily: $\mathbf{G}$ is a **sensitivity matrix**.

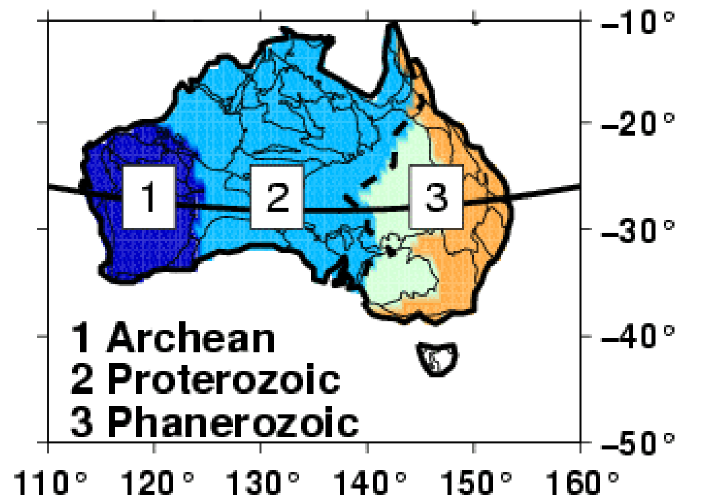


Resolution doesn't only depend on *path density*: many **criss-crossing** paths are needed. Modern global studies use hundreds of thousands of those.

[*Simons et al.*, 2002]

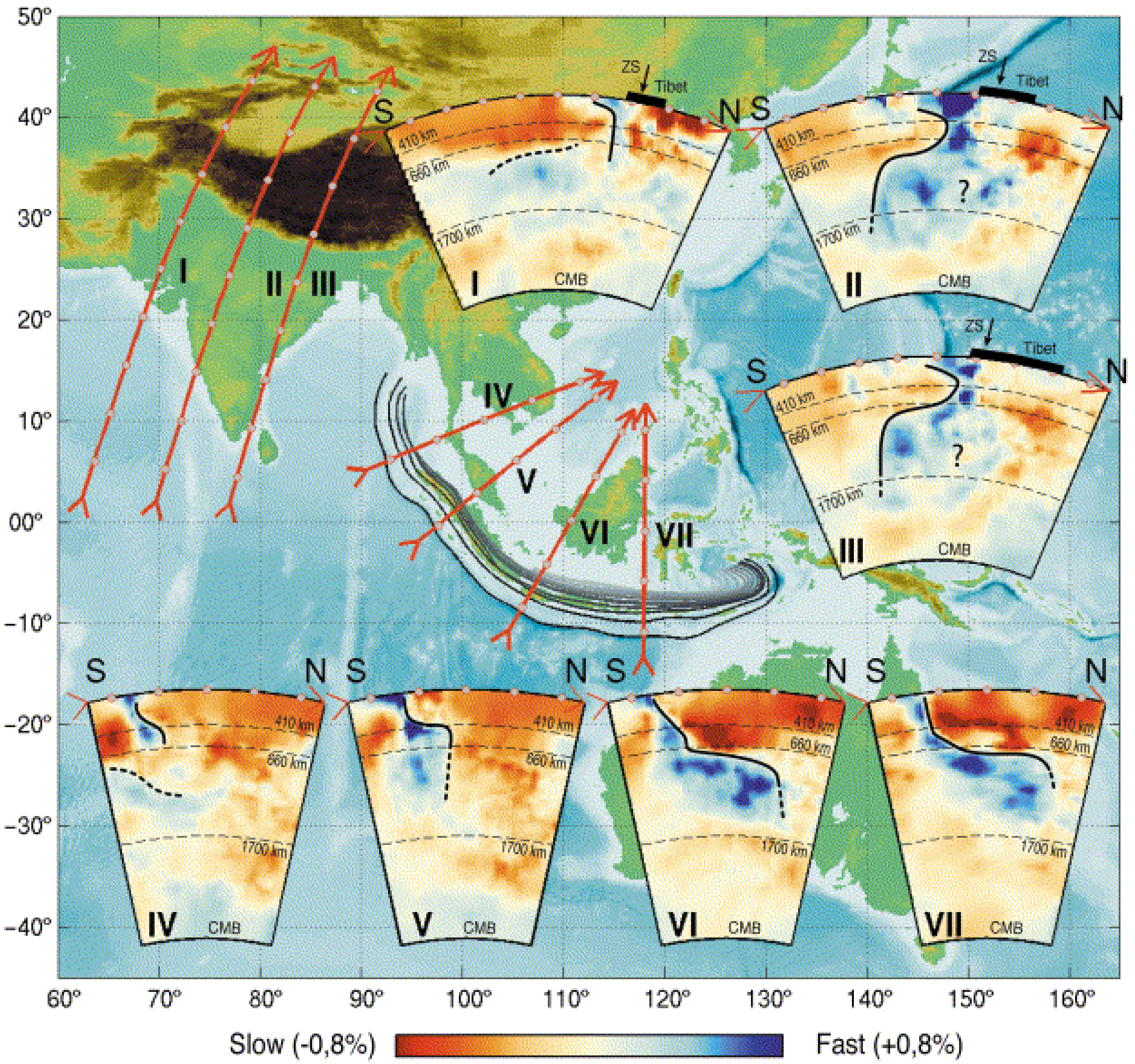
Regional models





[*Simons et al.*, 1999; *Fischer and van der Hilst*, 1999; *Simons et al.*, 2002]

Global models

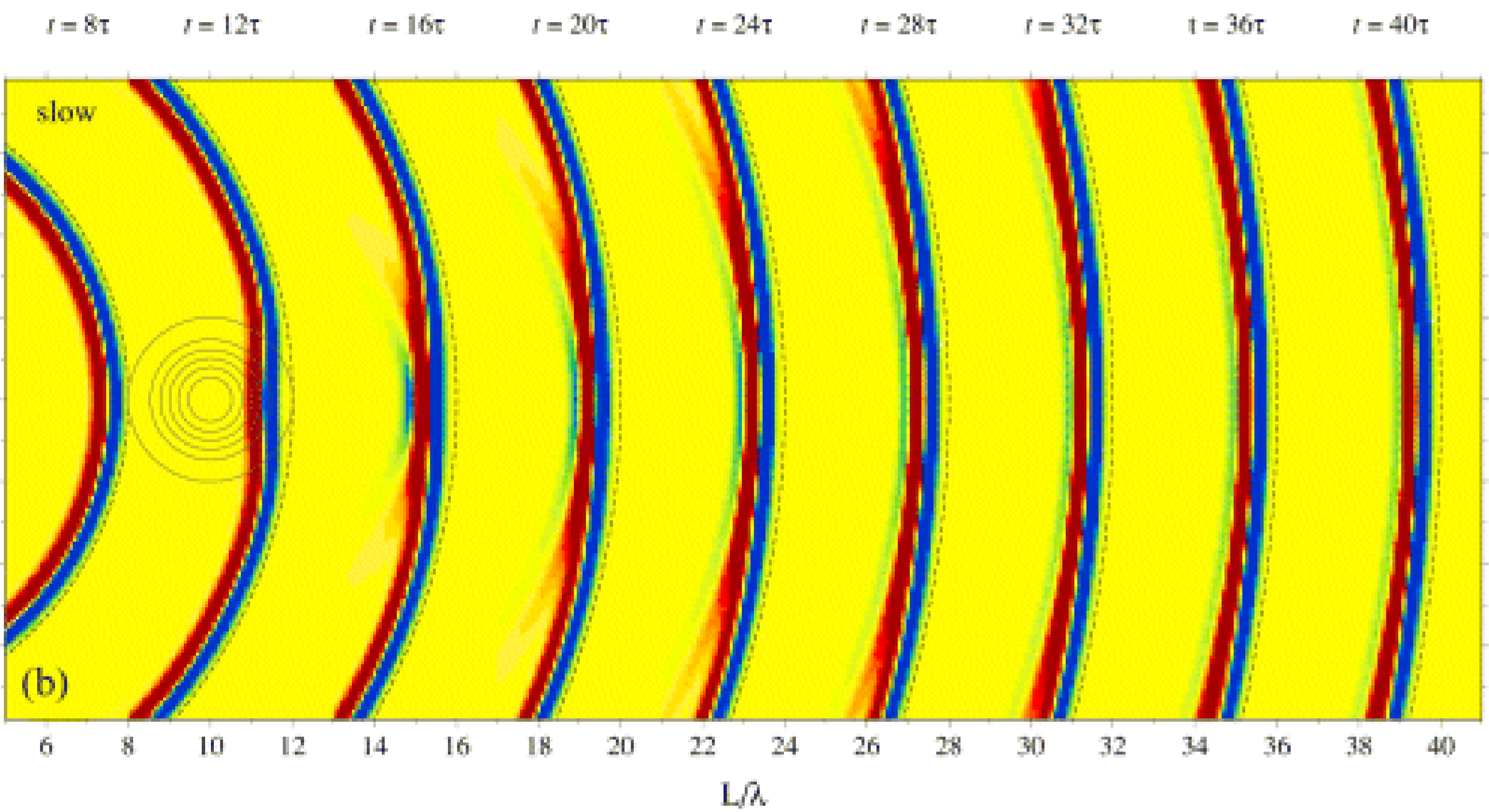


[*Replumaz et al.*, 2004]

Sensitivity: Obesity and wavefront healing

After discretization, parameterization, and regularization, every **geometrical ray** illuminates a **“fat tube”** in the model space.

But the **basic premise** — that a velocity anomaly sensed anywhere along the ray shows up as a travel-time anomaly at the receiver — is **wrong**. Wavefronts **heal**.



[*Hung et al.*, 2001]

Sensitivity: Infinite and finite frequencies

The following is only true when the wave is of an **infinitely high frequency**:

$$\delta t \approx \int_{\text{ray}} \left[-c_0^{-1}\right] \left(\frac{\delta c}{c_0}\right) dl. \tag{8}$$

Only at $\omega \longrightarrow \infty$ is the **sensitivity kernel** of the measurement δt to the model perturbation $\delta c/c_0$ given by c_0^{-1} exclusively *on the geometrical ray path*.

In reality, waves have a **finite frequency**, and measurements are at many different frequencies at that. The wave “feels” *off the ray*.

$$\delta t \approx \iiint_{\text{Earth}} K_{\delta t} \left(\frac{\delta c}{c_0}\right) dV. \tag{9}$$

Finding $K_{\delta t}$, a **3D Fréchet kernel**, is the name of the game — for now.

What are we measuring ? – 1

A **broadband travel-time anomaly** is the time shift that maximizes the **cross-correlation** of an observed seismogram, $u(t) = u_0(t) + \delta u(t)$, with the synthetic, $u_0(t)$, computed in the reference model:

$$\delta t = \arg\max \int_{t_1}^{t_2} u(t - \delta t)\, u_0(t)\, dt. \tag{10}$$

The waveform perturbation $\delta u(t)$ comes from perturbations in the **Earth model**:

$$\rho_0 \rightarrow \rho_0 + \delta\rho \quad \text{and} \quad \mathbf{C_0} \rightarrow \mathbf{C_0} + \delta\mathbf{C}, \tag{11}$$

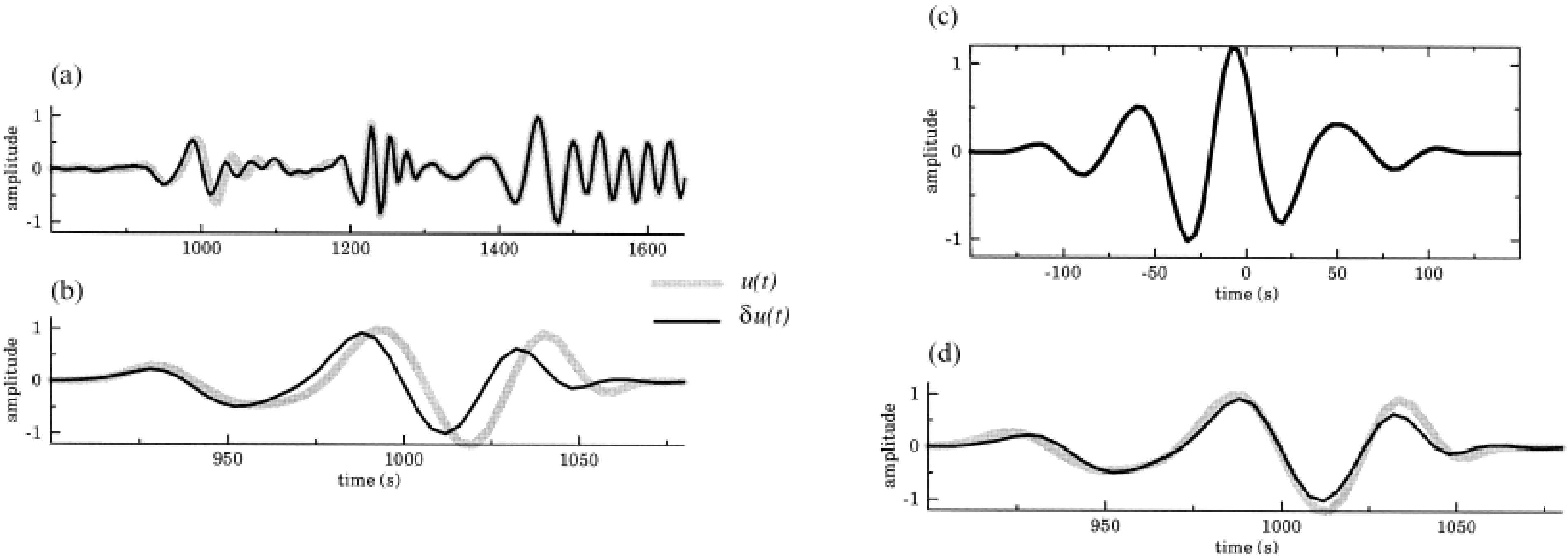
$$\mathbf{u_0} \rightarrow \mathbf{u_0} + \delta\mathbf{u}, \tag{12}$$

ρ density, $\mathbf{C}$ the elastic tensor, the above linearization the **Born approximation**.

The **seismogram** $u(t)$ is one component (vertical, radial, tangential) of the **wave-field** $\mathbf{u}(\mathbf{r}, \mathbf{t})$ measured at one particular location (the seismometer).

What are we measuring ? – 2

(a) **spherical-earth synthetic seismogram** and perturbed seismogram
(b) zoom on the **unperturbed** and perturbed *S* wave



(c) cross-correlogram of observed and synthetic seismogram
(d) alignment of **unperturbed** and perturbed seismogram after shift by δt

[*Marquering et al.*, 1999]

Two questions (only one multiple choice)

Question 1

How does the measurement δt depend on the waveform perturbation δu?

There is only one answer, and it has been known for a long time:

$$\delta t = \frac{\displaystyle\int_{t_1}^{t_2} \dot{u}_0(t)\,\delta u(t)\,dt}{\displaystyle\int_{t_1}^{t_2} \ddot{u}_0(t)\,u_0(t)\,dt} = \iiint_{\text{Earth}} K_{\delta t}\left(\frac{\delta c}{c_0}\right)\,dV. \tag{13}$$

Question 2

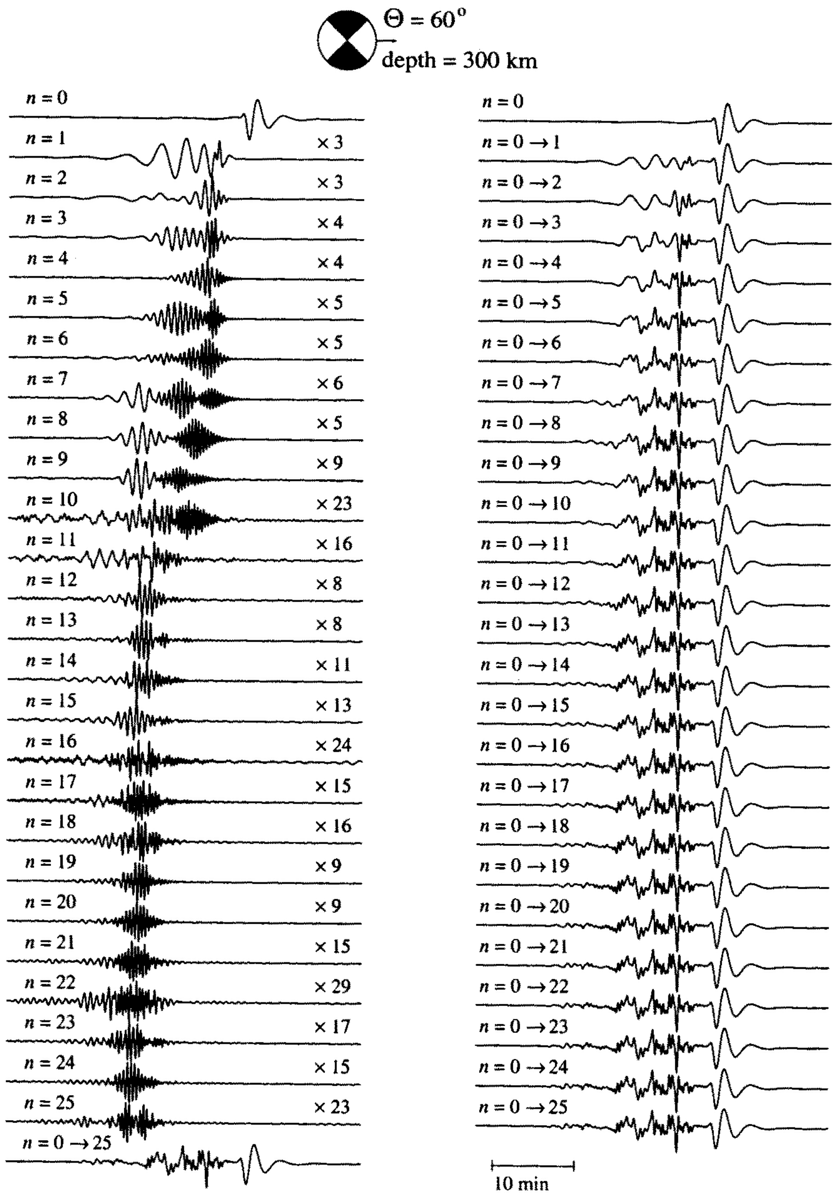
How does the waveform perturbation $\delta u(t)$ depend on $\delta\rho$ and $\delta\mathbf{C}$ of the Earth?

The answer depends on how the wavefield is computed.

This time there are several approaches, each with its own advantages.

[*Luo and Schuster*, 1991; *Zhao and Jordan*, 1998; *Marquering et al.*, 1999; *Dahlen et al.*, 2000; *Zhao et al.*, 2000]

Intermezzo I: Mode summation



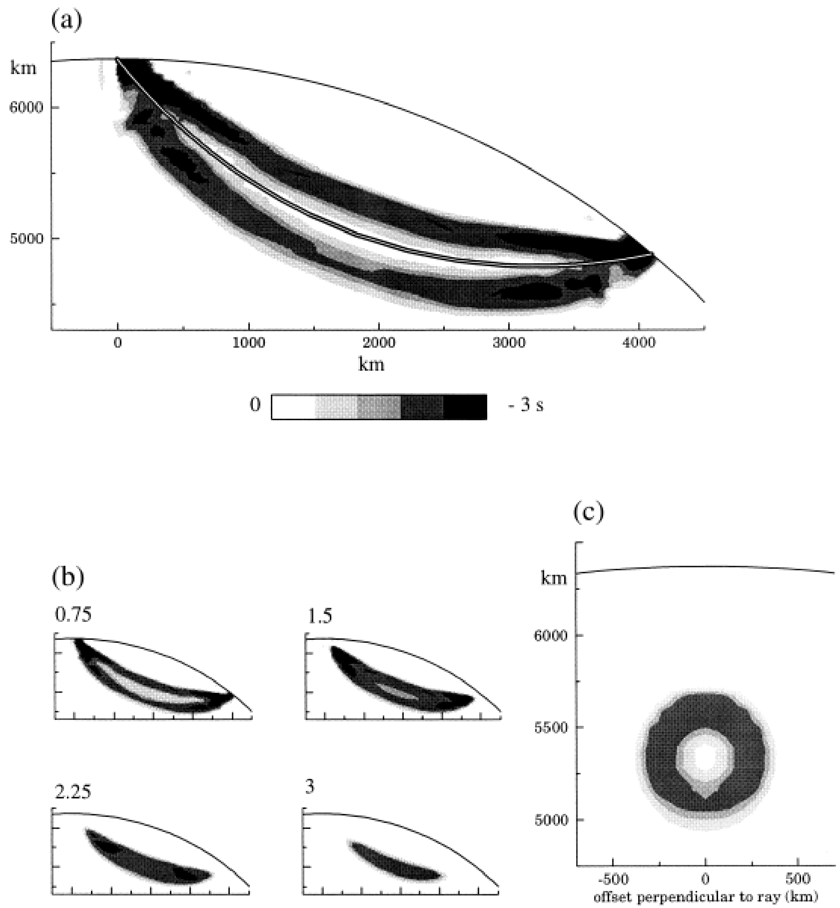
Every seismogram is a weighted sum of **normal modes** — eigensolutions to the wave equation. In radial Earth models, this is "easy", and to account for 3D perturbations, one considers their **coupling** (spheroidal, toroidal, etc...)

Surface-wave modes are also solutions to the wave equation. Whereas normal modes are *standing* waves that exist at "quantized" *degrees* and *orders* (think spherical harmonics), surface waves are *travelling* waves that can be calculated at equally spaced *frequencies*.

[*Dahlen and Tromp*, 1998]

Second approach to calculate δu

Make wavefield by **surface-wave mode summation** and consider their coupling.

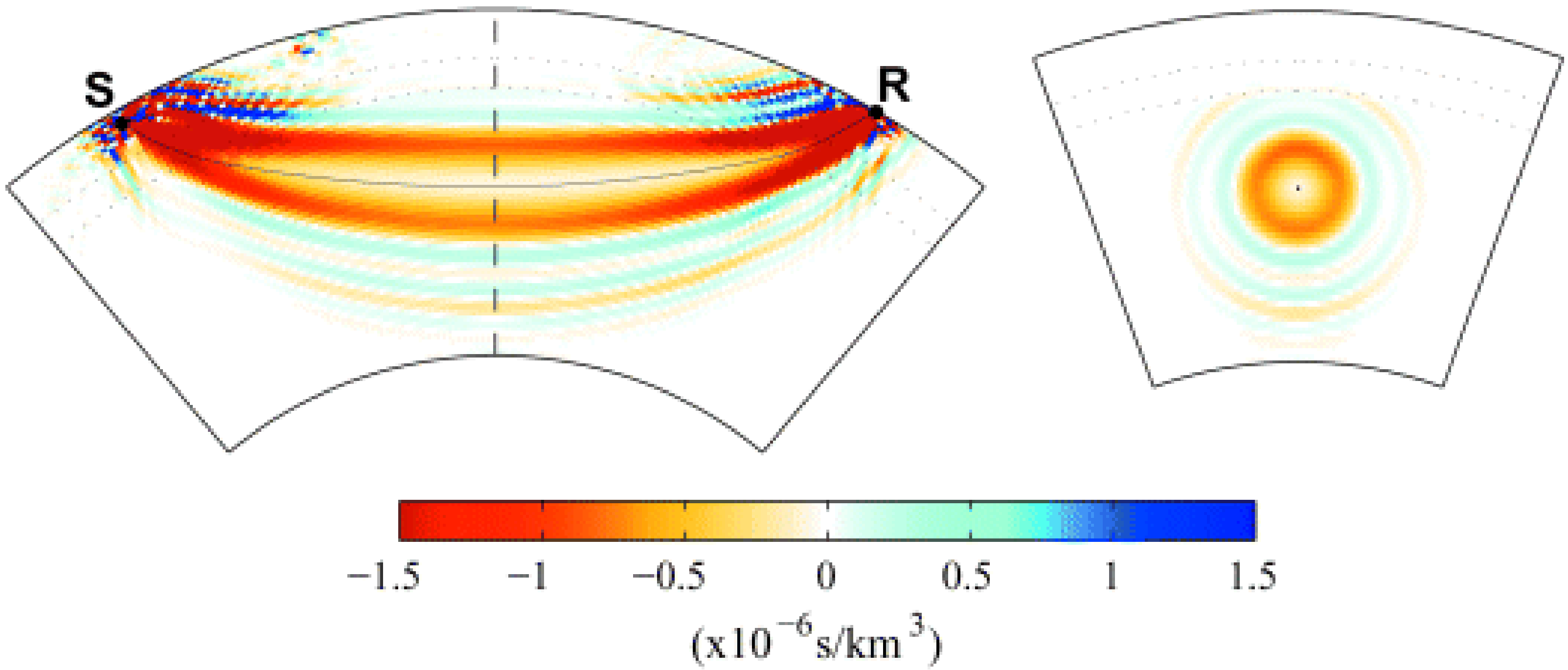


First appearance of the apt culinary metaphor **banana-donut kernel**.

[*Snieder and Nolet*, 1987; *Marquering et al.*, 1998, 1999; *Zhou et al.*, 2004; *Yoshizawa and Kennett*, 2005]

First approach to calculate δu

Compute the wavefield by **normal-mode summation** and take into account the **mode coupling** due to aspherical perturbations.



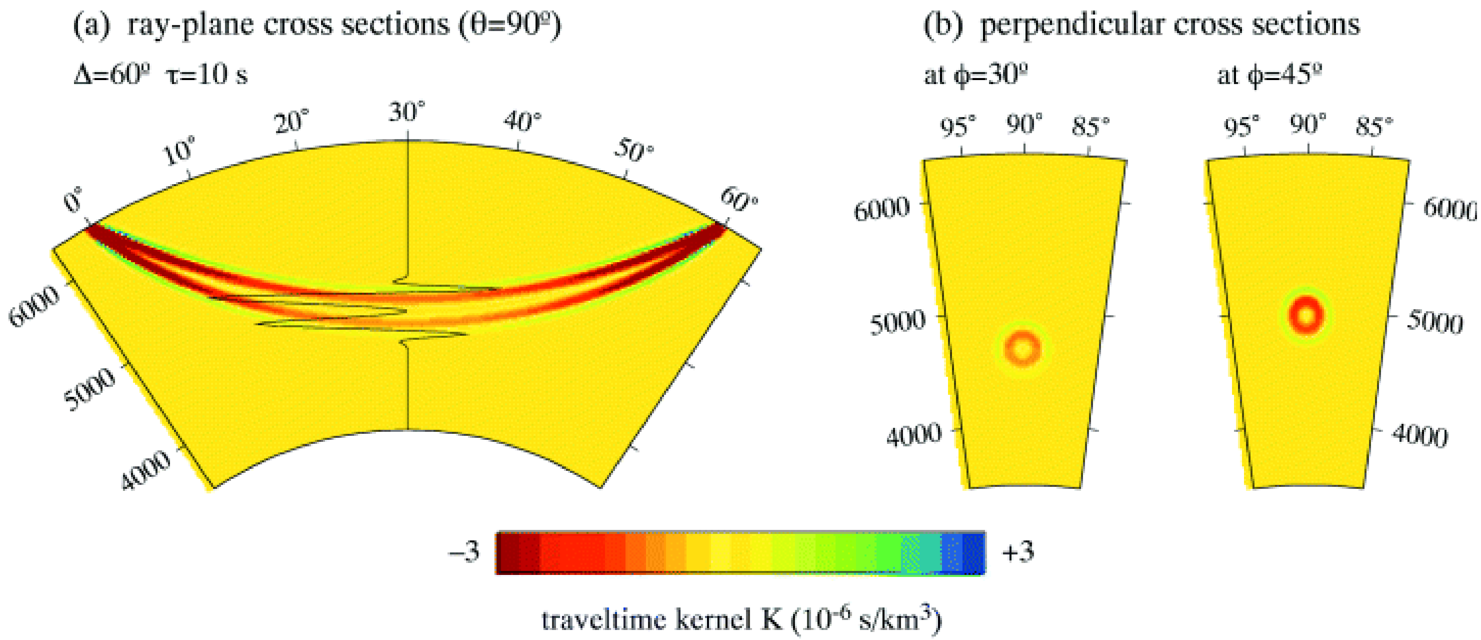
Normal-mode theory is **complete** but cumbersome numerically.

[*Zhao et al.*, 2000; *Capdeville*, 2005]

Third approach to calculate δu

Ray theory is dead. Long live ray theory!

No more mode sums; use **ray sums**. Assume **singly-scattered** waves (all types).



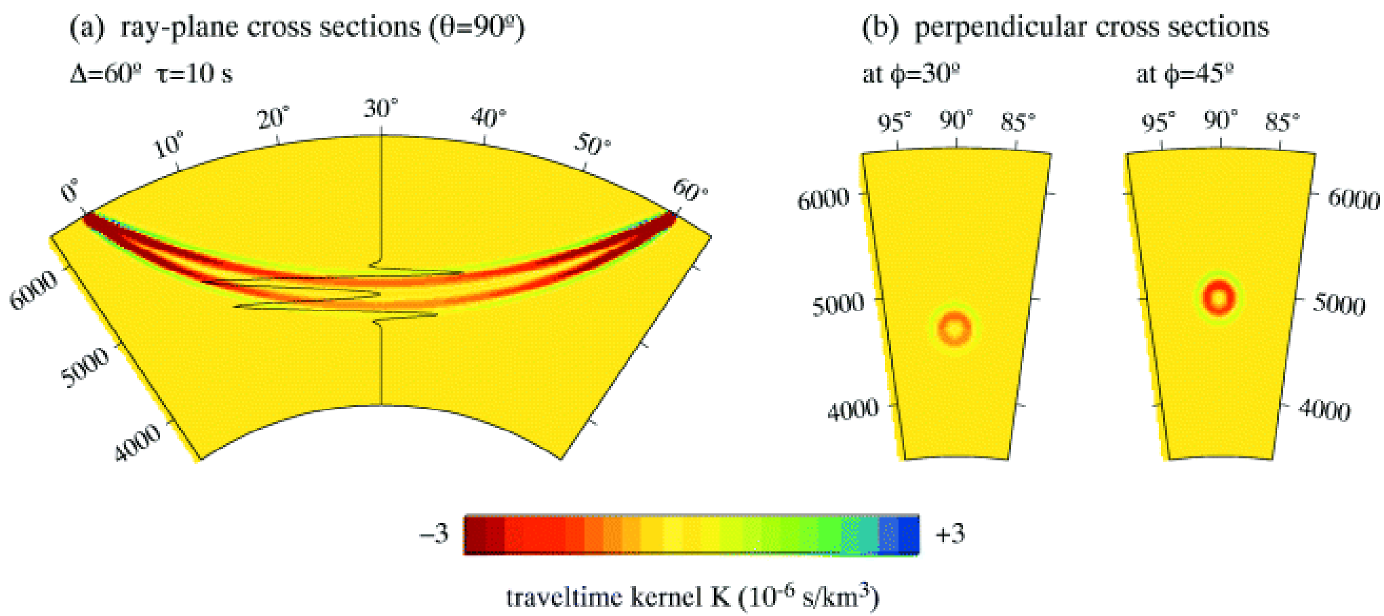
This is more efficient than mode-coupling but requires **lots of ray-tracing**: need to trace *all possible* rays from the source to every point in the Earth, and of *all possible* rays from those points to the receiver. It breaks down for the most complex phases.

[*Zhao and Dahlen*, 1996; *Dahlen et al.*, 2000; *Zhao et al.*, 2000; *Hung et al.*, 2000]

Fourth approach to calculate δu

The paraxial approximation, and the most widely used method today.

No more multiple ray tracing; trace only the **geometrical ray**; expand travel-time surface about it; only consider **like-type scattering** in the vicinity of the central ray.



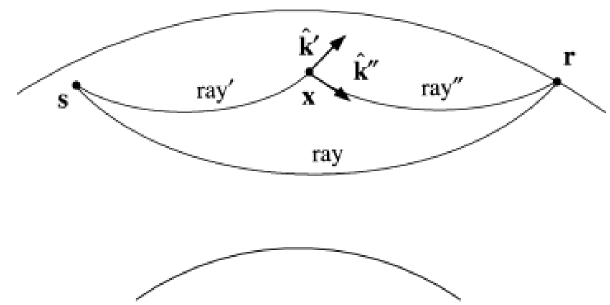
This is **much more efficient** than the previous methods, but it breaks down somewhat earlier. However, the approximations are justifiable for common phases such as *P*, *PcP*, *PP*, *S*, *ScS*, *SS* between 30° and 90° distance.

[*Dahlen et al.*, 2000; *Hung et al.*, 2000; *Yoshizawa and Kennett*, 2002]

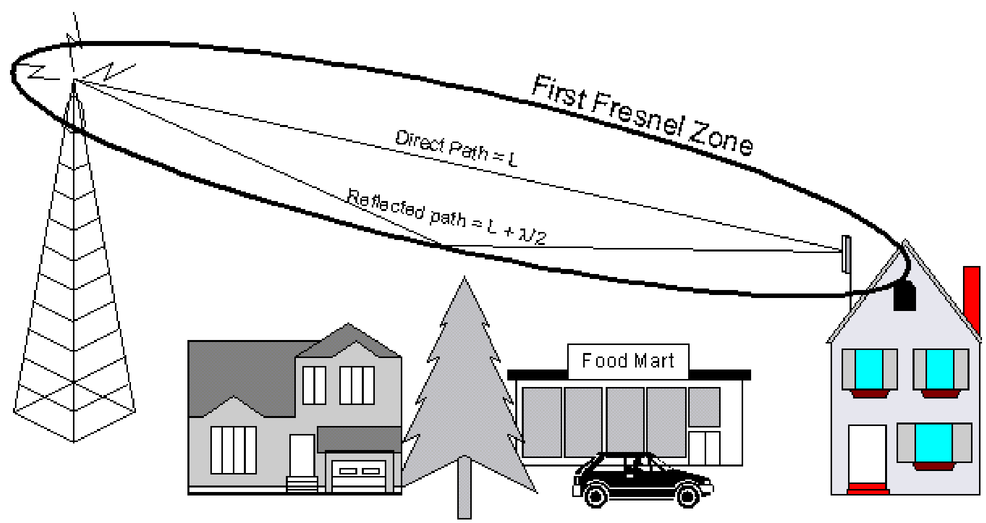
Why does this work at all ?

The sensitivity kernel $K_{\delta t} \approx 0$ outside of the **first Fresnel zone**:

$$0 \leq \bar{\omega}(T' + T'' - T) \leq \pi. \qquad (14)$$



Outside of the Fresnel zone, destructive interference between nearby frequencies kills the sensitivity! This has been known since... well, Fresnel's time (1820s).



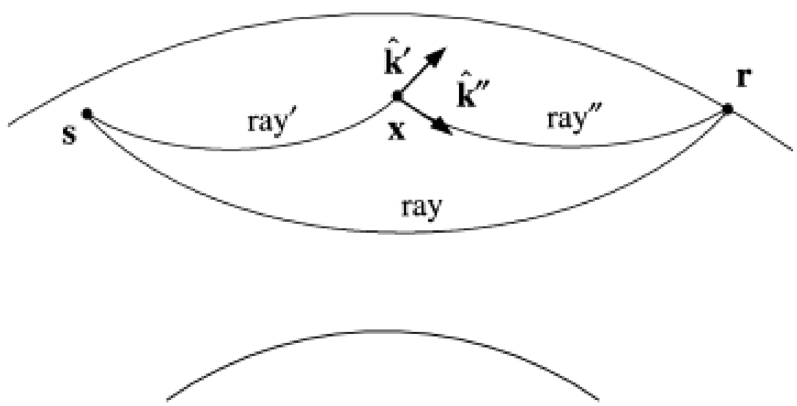
[*Dahlen et al.*, 2000; *Nolet et al.*, 2005]

Worried about the hole ?

Think of the 3D Born kernels as pictures of travel-time perturbations with respect to the travel-time of the unperturbed geometrical (Fermat) ray. The perturbations are due to **scatterers off** the central ray.

Propagation from the source to a *scatterer on* the central ray — and from there on to the receiver **defines** the central ray — there is **no way** that this generates a *cross-correlation* travel-time shift.

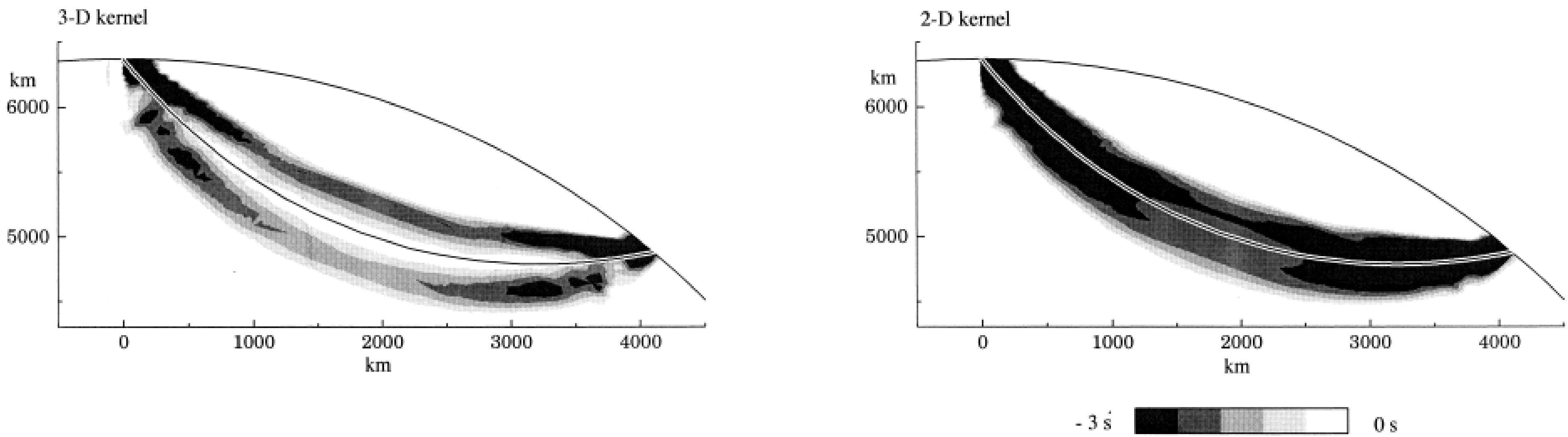
The situation is different for amplitudes — and it sure is **counterintuitive**.



[*Nolet et al.*, 2005]

Reduction from 3D to 2D

Long the source of heated debate... In 2D, the donut hole *actually* disappears...



It's like saying the perturbations are cylindrically symmetric about the ray plane...

And what a strange Earth that would be...

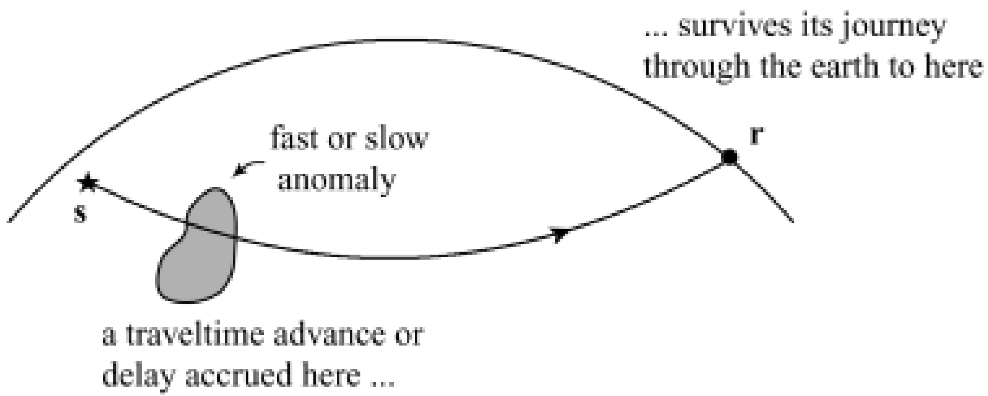
[*Zhao and Jordan*, 1998; *Marquering et al.*, 1999; *Zhao et al.*, 2000]

Reduction from 3D to 1D

The area of the donut is $-1/c$. If the wavelength of the wave is small compared to the scale length of the heterogeneity, we simply get ray-theory back:

$$\iiint_{\text{Earth}} K_{\delta t} \left(\frac{\delta c}{c_0} \right) dV \rightarrow \int_{\text{ray}} \left[-c_0^{-1} \right] \left(\frac{\delta c}{c_0} \right) dl. \tag{15}$$

Thus, finite-frequency banana-donuts provide the **natural extension** of linearized infinite-frequency ray theory.



[*Zhao and Jordan*, 1998; *Marquering et al.*, 1999; *Zhao et al.*, 2000]

Fourth approach: Practical aspects

It is the fourth approach that is commonly referred to as *the* **banana-donut** theory.

To recapitulate, it [1] **measures** δt by **cross-correlation**, it [2] **reduces** the sensitivity of δt for *P* (or *S*) "travel times" to perturbations in the *P* (or *S*) wave speeds (only), via **3D Fréchet kernels**, as

$$\delta t \approx \iiint_{\mathrm{Earth}} K_{\delta t} \left(\frac{\delta c}{c_0} \right) \, dV, \tag{16}$$

and [3] to **compute** $K_{\delta t}$ it uses **dynamic ray tracing** of the geometrical ray **only**.

Subsequently, a linear inverse problem in the sense $\mathbf{G} \cdot \mathbf{m} = \mathbf{d}$ is set up, where $\mathbf{d}$ contains the (cross-correlation) travel times δt, $\mathbf{m}$ is **some model parameterization** and $\mathbf{G}$ contains the above kernels **in the same model space basis**.

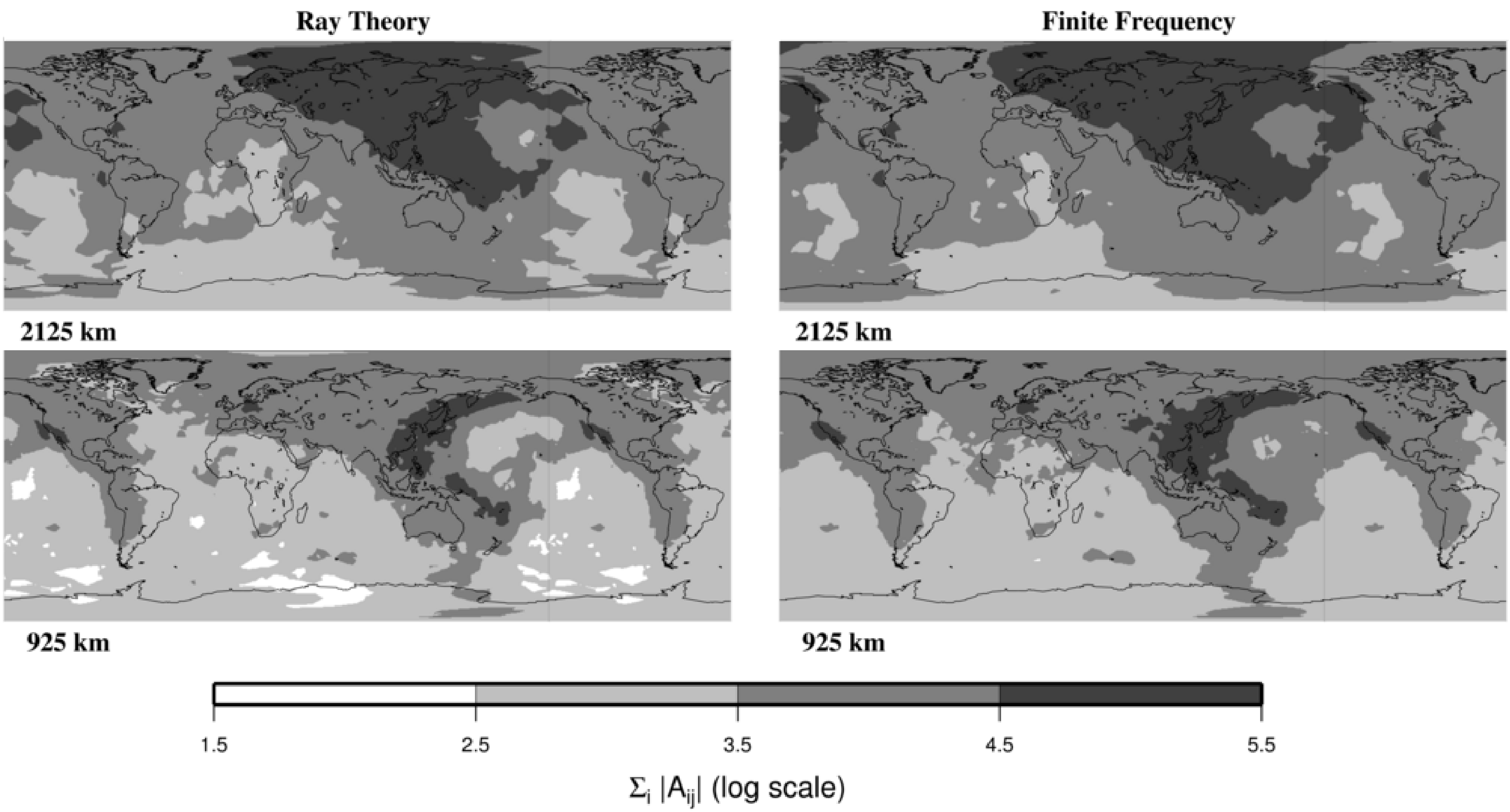
And **then** the inversion is performed by a **human** being, with **regularization**.

[*Montelli et al.*, 2004a, b; *Nolet and Montelli*, 2005; *Nolet et al.*, 2005]

Banana-donut inversions: Enhanced sensitivity

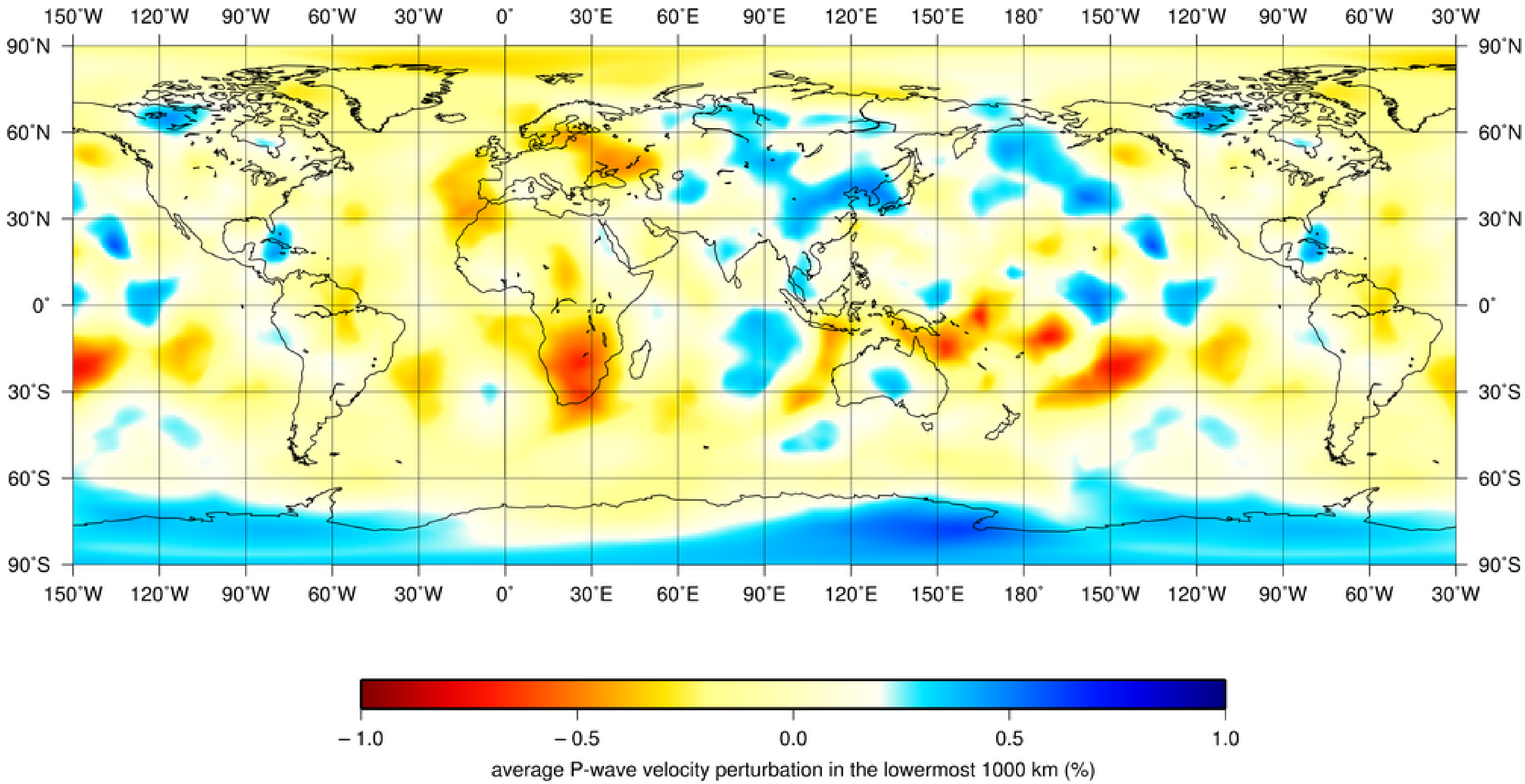
The banana-donut sensitivity **fattens the rays**.

Much like ray theory would under a coarse parameterization, but **motivated by physics**, not simply due to numerical discretization and regularization.



[*Montelli et al.*, 2004b]

Banana-donut inversions: Plumes. Finally ?



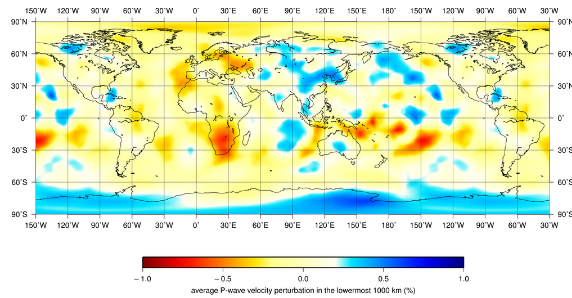
Seems like this has been a real bone of contention...

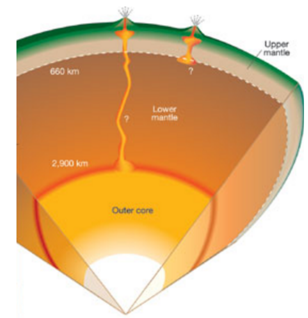
[*Montelli et al.*, 2004a]

Lost in the null space ?

All the *theory* is — basically — **non-controversial** and about as widely accepted as the theory of evolution. The bananas are yummy, people had to get used to the donut hole... but they're there **to stay**.

It's the **arrows** in the following that most people have difficulty digesting:

 +  → 

→ 

Those reflect the fact that seismic tomography is, after all, still an **art**...

unfortunately, **more Pollock than Hopper**.

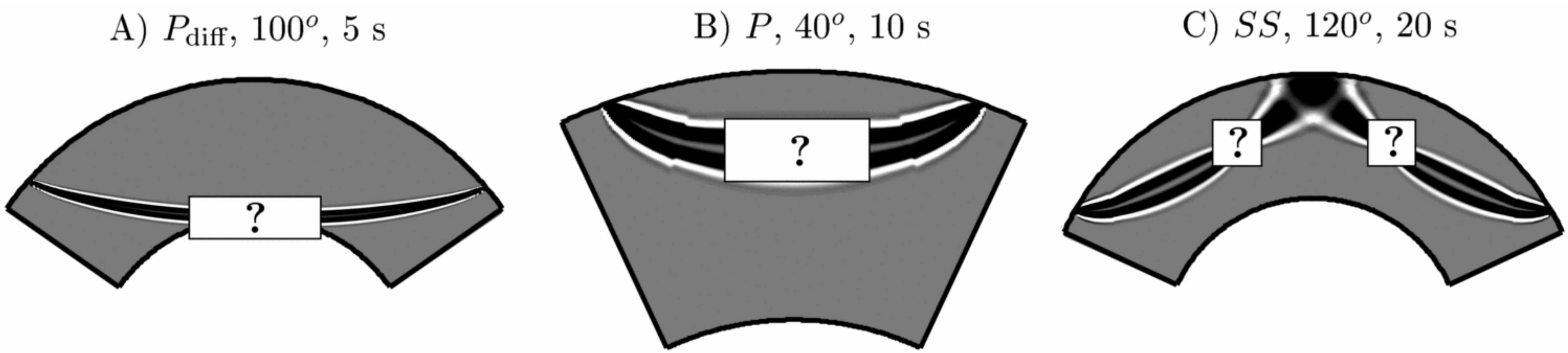
Might there be another way out?

[*de Hoop and van der Hilst*, 2005; *van der Hilst and de Hoop*, 2005; *Boschi*, 2006; *Boschi et al.*, 2006; *Trampert and Spetzler*, 2006]

Why a *fifth* approach is needed

Approaches 1–3 (normal modes, surface-wave modes, complete ray sums) are numerically burdensome, i.e. **infeasible** in tomographic practice.

Approach 4 (the paraxial approximation) is **inapplicable** in the vicinity of any critical refraction, diffraction, or caustic...



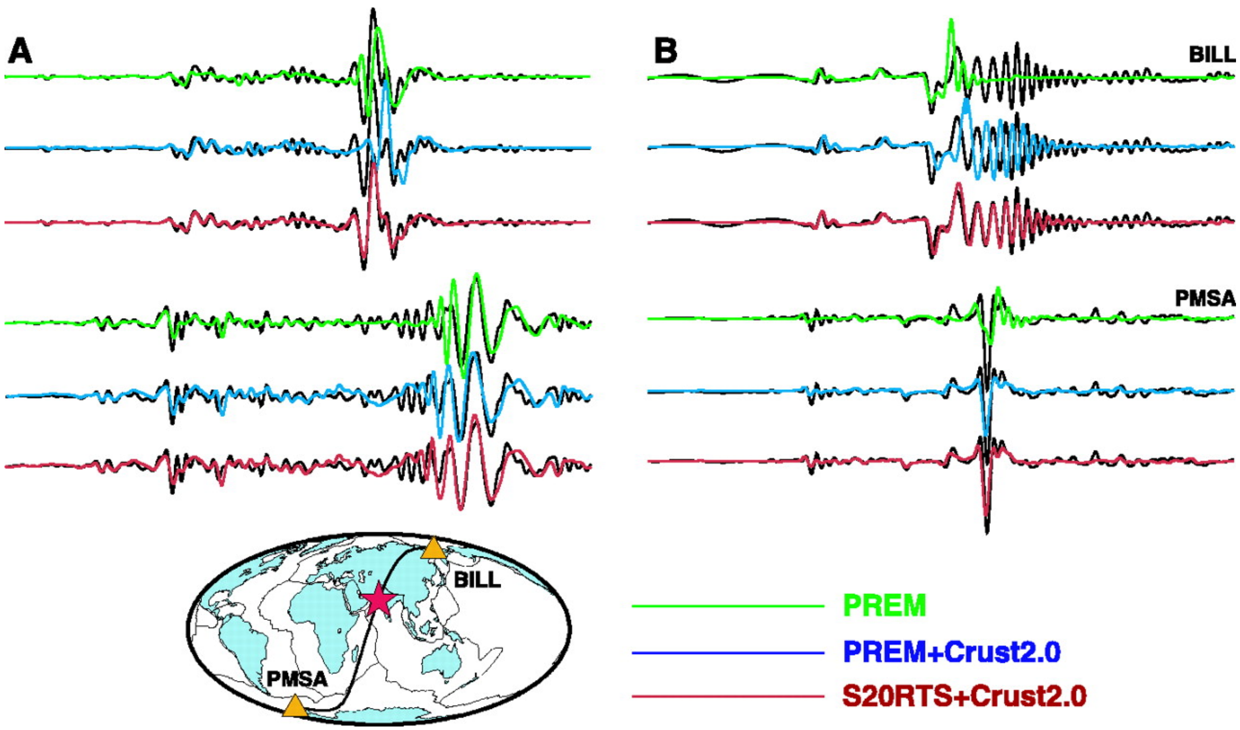
Look for one more way to compute $\delta u(t)$ as a result of Earth model perturbations.

The answer is to go via **fully numerical solutions** to the wavefield.

[*Nissen-Meyer et al.*, 2007a]

One of the most powerful contemporary **grid-based methods** to produce **synthetic seismograms** in **realistic** 3D media (e.g. self-gravitating, rotating, anisotropic, attenuative, heterogeneous Earth models).

Combines the **geometrical flexibility** of the finite-element method with the **exponential convergence** and **weak numerical dispersion** of spectral methods.



[*Komatitsch and Vilotte*, 1998; *Komatitsch and Tromp*, 1999, 2002a, b; *Komatitsch et al.*, 2002; *Fournier et al.*, 2004]

Fifth approach: Born again

Modern (e.g. SEM) methods can compute wavefields in *arbitrary* 3D background models. We no longer have to assume that only *P* (or *S*) wave speed perturbations influence *P* (or *S*) cross-correlation travel times of *P* (or *S*) waveforms.

We can take one step back and restart from the **Born approximation** (11–12):

$$\delta u(t) = \iiint\limits_{\mathrm{Earth}} \left\{ K_{\delta\rho}(t) \left(\frac{\delta\rho}{\rho_0} \right) + K_{\delta\mathbf{C}}(t) \left(\frac{\delta\mathbf{C}}{\mathbf{C}_0} \right) \right\} dV, \tag{17}$$

where computing **3D waveform kernels** involves **one forward simulation** and **one backward simulation** and their interaction by **convolution**:

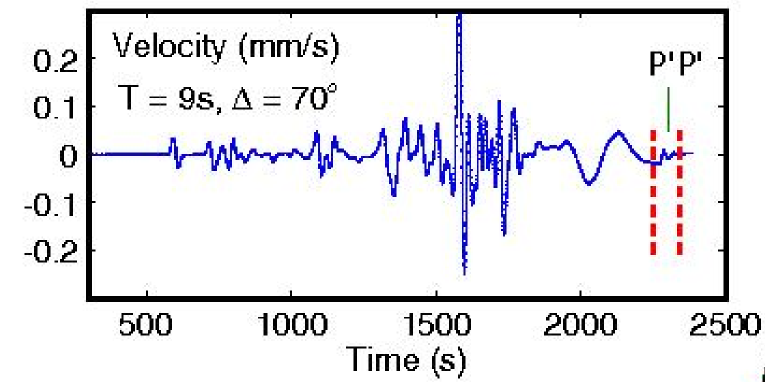
$$K_{\delta\rho}(t) = -\int_0^t \dot{u}_i^{\mathrm{to}}(\tau)\, \dot{u}_i^{\mathrm{fro}}(t-\tau)\, d\tau, \tag{18}$$

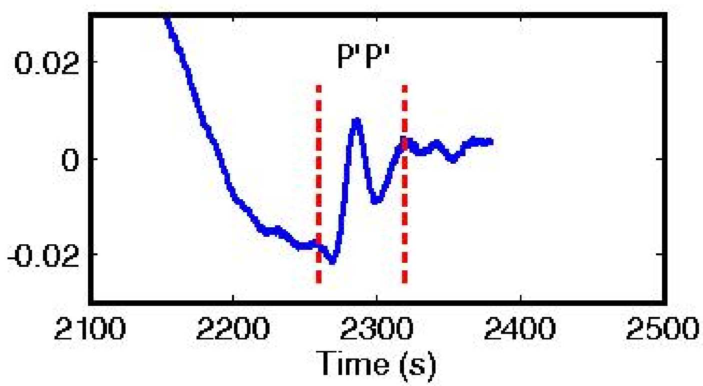
$$K_{\delta\mathbf{C}}(t) = -\int_0^t \epsilon_{ij}^{\mathrm{to}}(\tau)\, \epsilon_{kl}^{\mathrm{fro}}(t-\tau)\, d\tau. \tag{19}$$

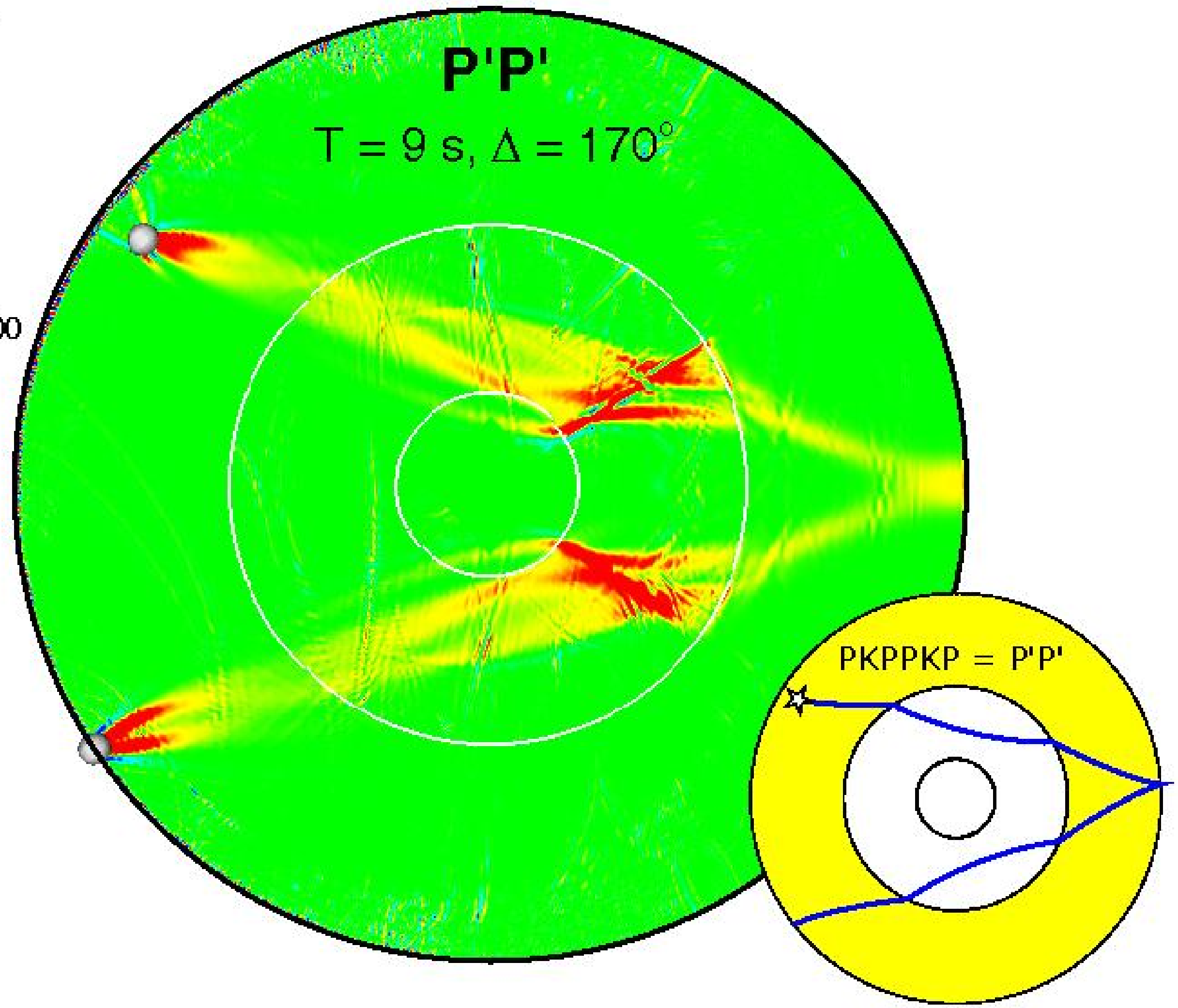
[*Marquering et al.*, 1998; *Tromp et al.*, 2005; *Nissen-Meyer et al.*, 2007a, b; *Tape et al.*, 2007]

Fifth approach: Have you seen my *phase* ?

Different flavors of SEM wavefield computation can be used...







[*Tromp et al.*, 2005; *Nissen-Meyer et al.*, 2007a, b; *Tape et al.*, 2007]

Extensions: Anything goes

The **3D Born waveform kernels** are the basic **building blocks** with which the sensitivity of *anything* (some observable waveform attribute) to the perturbation of *anything* (some Earth parameter) can be constructed.

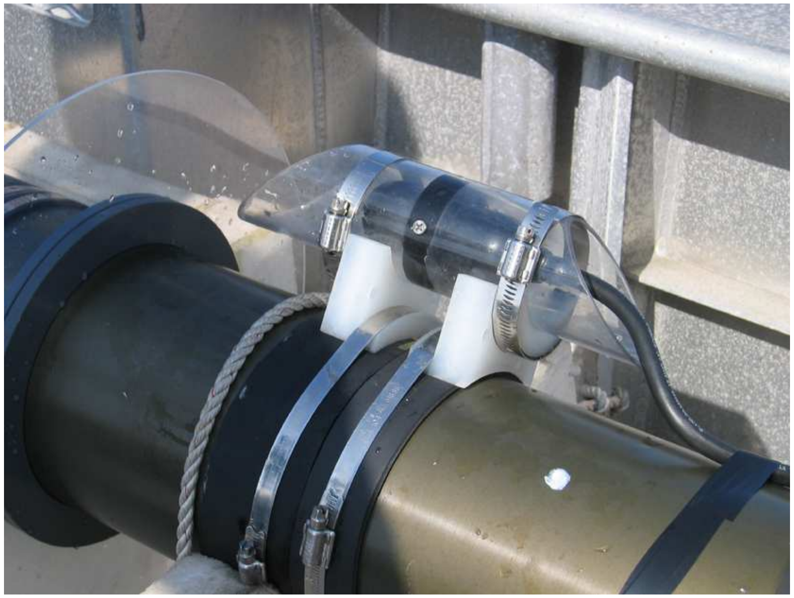
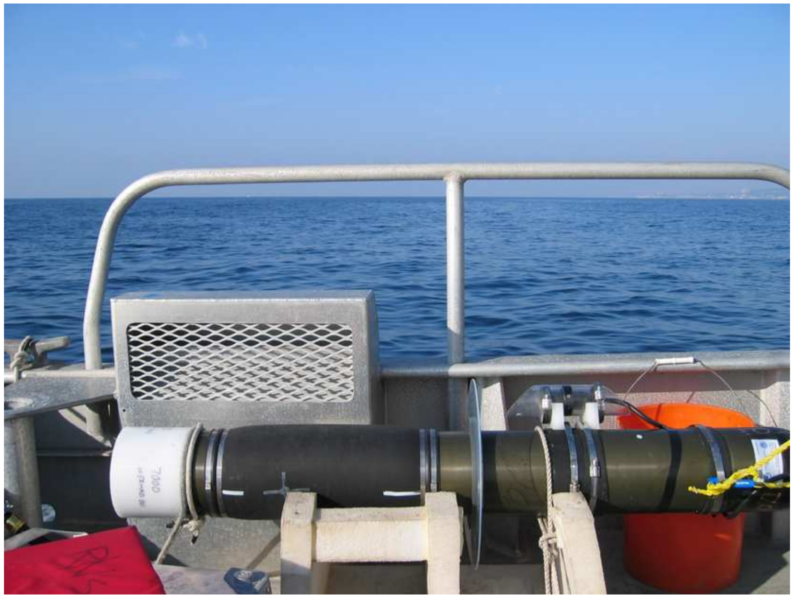
With the fifth approach we had gone back to square one and constructed cross-correlation travel-time measurements. But we can think of Fréchet kernels for **amplitudes**, **boundary undulations**, **attenuation**, **shear-wave splitting**, **rms waveform misfits**... if you can name it, someone will make it!

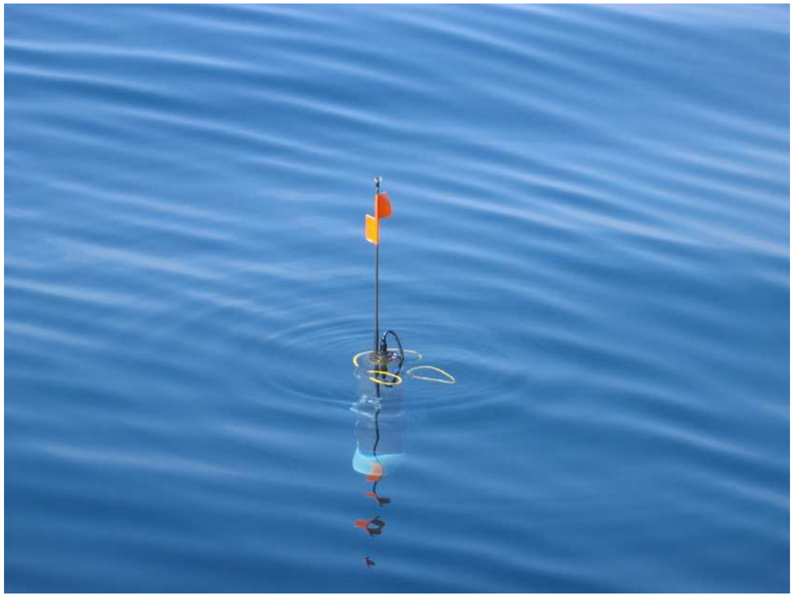
A combination of some of the above approaches are currently being implemented to enable **full waveform tomography**... the F · u · t · u · r · e ! You'll hear about are **adjoint methods** (non-linear, 3D SEM-based, *very* expensive) and **3D-to-2D methods** (linear, axisymmetric SEM-based, with numerical shortcuts).

Is there anything simpler?

[*Dahlen and Baig*, 2002; *Favier and Chevrot*, 2003; *Dahlen*, 2005; *Tromp et al.*, 2005; *Dahlen and Zhou*, 2006; *Sigloch and Nolet*, 2007]

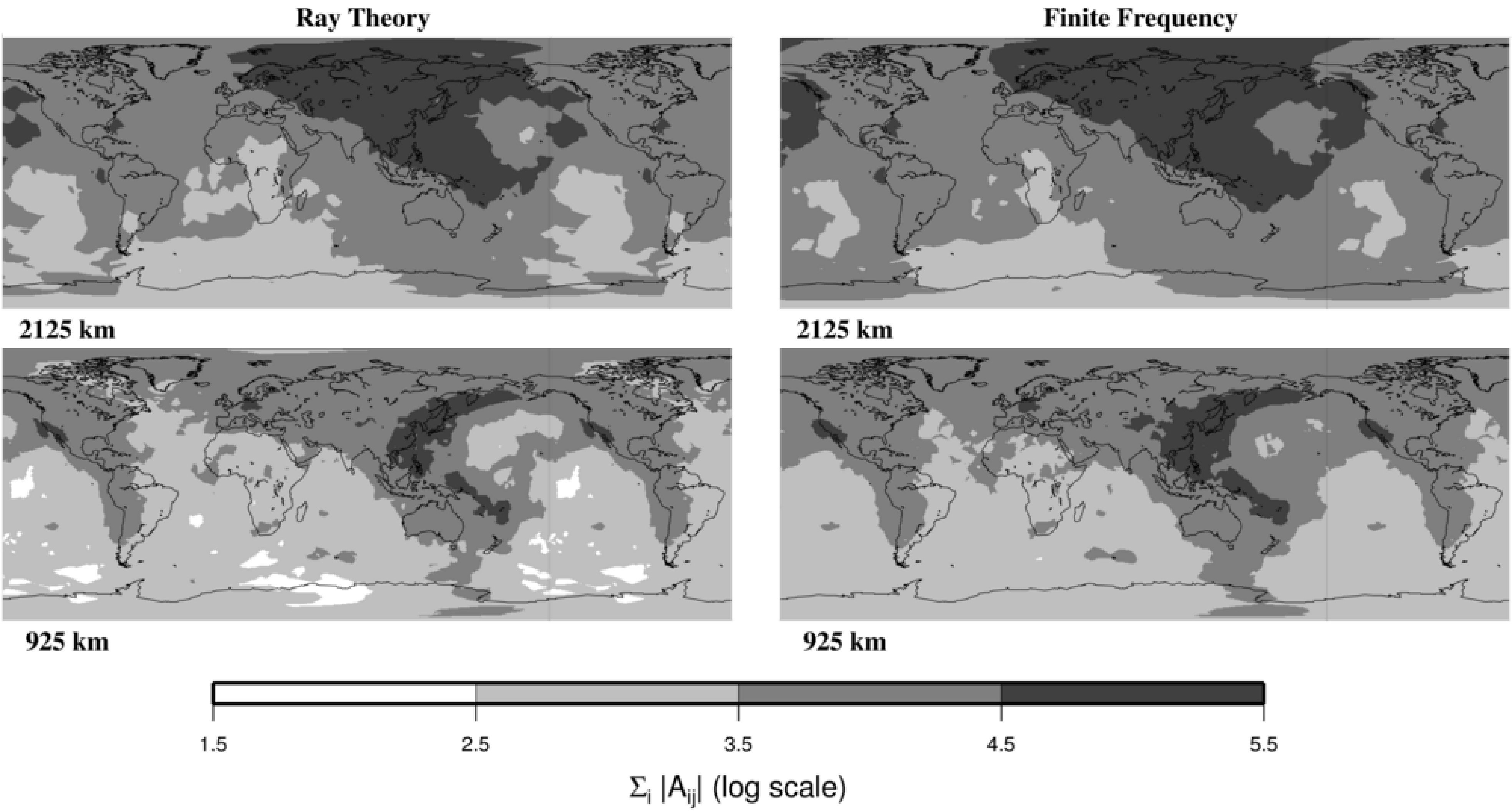
A future for drifting seismic networks – 1



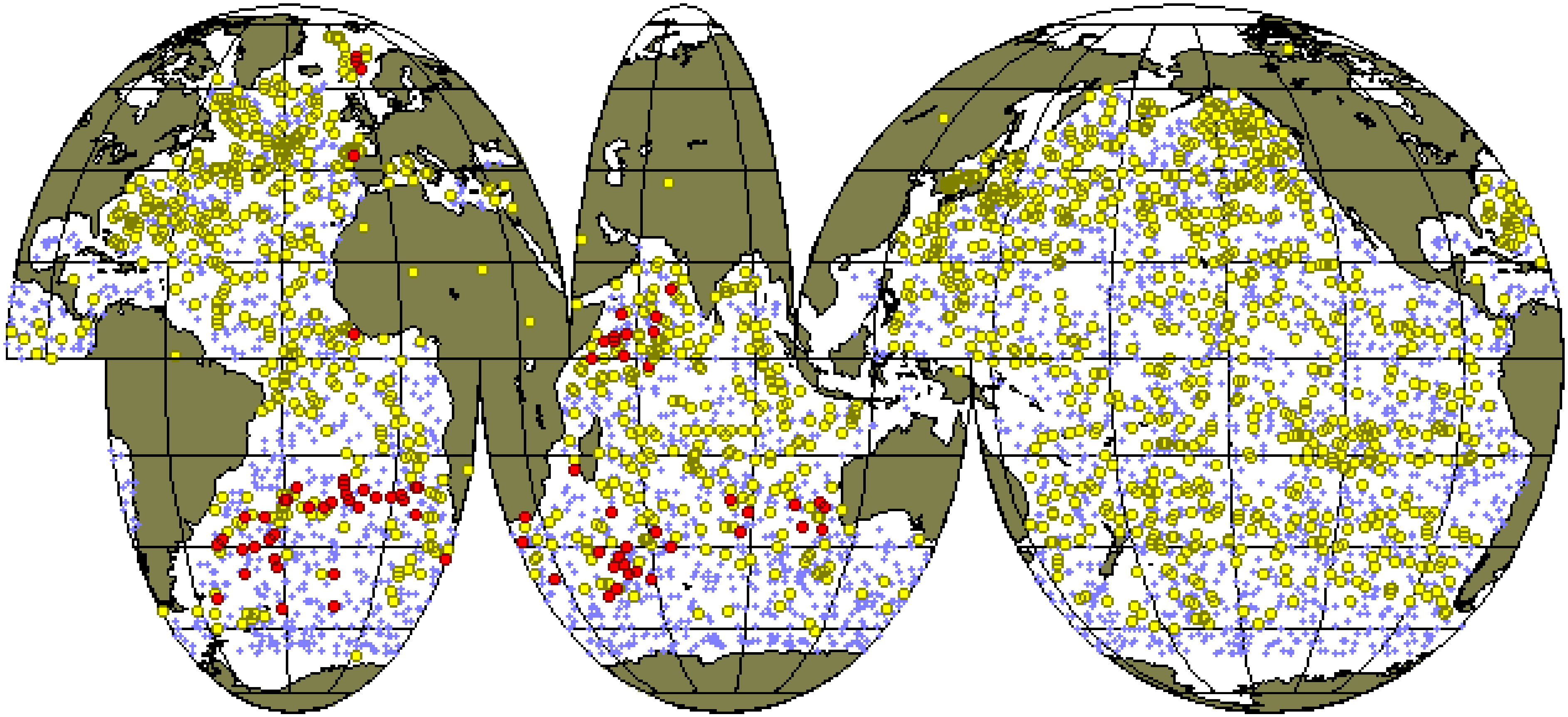
[*Simons et al.*, 2006]

Enhanced sensitivity by adding data points



[*Montelli et al.*, 2004b]

A future for drifting seismic networks – 2



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